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A SETAR Model for Taiwan Stock Exchange Capitalization Weighted Stock Index: Non-linearities and Forecasting Comparisons

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Abstract

In this paper, we analyze the change in structure which occurred in Taiwan stock index, while finding a better non-linear model. We examine the out-of sample performance of non-linear time series SETAR model by employing Taiwan Stock Exchange Capitalization Weighted Stock Index over the period from January 3, 2005 to December 31, 2009. Furthermore, we do the unit root test before the model setting and then compare the out-of-sample forecasting performances between standard linear ARIMA model and non-linear SETAR model. Empirically, we find that non-linear SETAR model has superior forecasting power than linear ARIMA model does in Taiwan stock market.

Keywords: Taiwan Stock Exchange Capitalization Weighted Stock Index, SETAR Model, Threshold Model, Forecasting

1. Introduction

1.1 Background and Motivation

The financial tsunami is a crisis that happened in 2007. It broke out in the United States, and then spread to the whole world. The fluctuation is manifest in Taiwan stock index. It has been even dramatically losing 60% of the market value within six months. During such a drastic change, economic systems might also result in changes of stock prices to some extent.

However, in the traditional model, researchers use the linear model in forecasting the behavior of stock prices. It is interesting to examine their effects on the regime-swift issue in stock prices after the recent financial crisis. Therefore, we judge the non-linear model and the linear model by using the out-of sample forecasting abilities. Hopefully, we could obtain the better forecasting model for the possibility of the non-linearities of the stock prices.

1.2 Thesis Structure

In the beginning, to build up a SETAR model, we need to determine a switching point. We check stationarity by using unit root test. In the model selection phase, we adopt box and Jenkins's method to build up the SETAR and ARIMA model. Finally, we check the out-of-sample forecasting power of these two models and find a comparatively better one.

The remainder of the paper is organized as follows. Section 2 presents a literature review. Section 3 introduces empirical models. Empirical results are discussed in section 4. Finally, section 5 is the conclusion.

2. Literature Review

As in the master of TAR research, Tong's publication presented. TAR model were initially proposed by Tong (1978) and Tong and Lim (1980) at an Ordinary Meeting of the Royal Statistical Society meeting. The threshold idea was thus conceived in 1977 and Tong put the idea into practice which meant a huge amount of computer experimentation. The first presented was the SETAR (Self-exciting threshold autoregressive) model. Then it became more general in the further researches.

Tasy (1989) carried on suggesting a simple yet widely applicable model-building procedure for threshold autoregressive models and a test for threshold nonlinearity. Then LeBaron (1992) demonstrated that different levels of volatility can be regarded as the regime-determining process. One year later, Kräger and Kugler (1993) argued that exchange rates might show regime-switching behavior and found that the significant threshold effects, estimated by SETAR models, affected the exchange rates for five currency exchange rates. Till the year of 1998, more econometricians put their

attention on the ergodicity/stationarity problem. De Gooijer (1998) considered regime-switching to the MA model and used validation criteria on SETAR model selection.

Clements and Smith (2001) evaluated forecasts from SETAR models of exchange rates and compared them with traditional random walk measures. Hansen (2001) used Chow test in testing unknown structural change timing. Boero and Marrocu (2002) showed clear gains from the SETAR model over the linear competitor, on MSFEs evaluation of point forecasts, in sub-samples characterized by stronger non-linear models. Boero (2003) studied the out-of-sample forecast performance of SETAR models in Euro effective exchange rate. The SETAR models have been specified with two and three regimes, and their performance has been assessed against that of a simple linear AR model and a GARCH model. Kapetanios and Yongcheol (2006) distinguished a unit root process from a globally stationary three-regime SETAR process.

An ARIMA model can be considered as a special type of regression model-in which the dependent variable has been stationarized and the independent variables are all lags of the dependent variable and/or lags of the errors. In this study, we quote Box-Jenkins approach to modeling ARIMA processes which was announced by Box and Jenkins in 1970. An ARIMA process is a mathematical model used for forecasting. Box-Jenkins modelling involves identifying an appropriate ARIMA process, fitting it to the data, and then using the fitted model for forecasting. One of the attractive features of the Box-Jenkins approach to forecasting is that ARIMA processes are a very rich class of possible models and it is usually possible to find a process which provides an adequate description to the data. All these years, ARIMA forecasting models for ecomomic variables were broadly developed, estimated, and then used for ex-post and ex-ante forecasts.

3. Empirical Models

The main purpose of the present study is to compare the out-of-sample forecast performance of the non-linear SETAR model to the linear ARIMA model in the Taiwan Stock Exchange Capitalization Weighted Stock Index. The ARIMA model is specifically designed to make forecasts based on the values of a time series. It was first developed in the late 60s but was systemized by Box and Jenkins in 1976. Here in this paper, we use steps presented by Box and Jenkins (1976). The description of the estimation steps could graphed as follows.

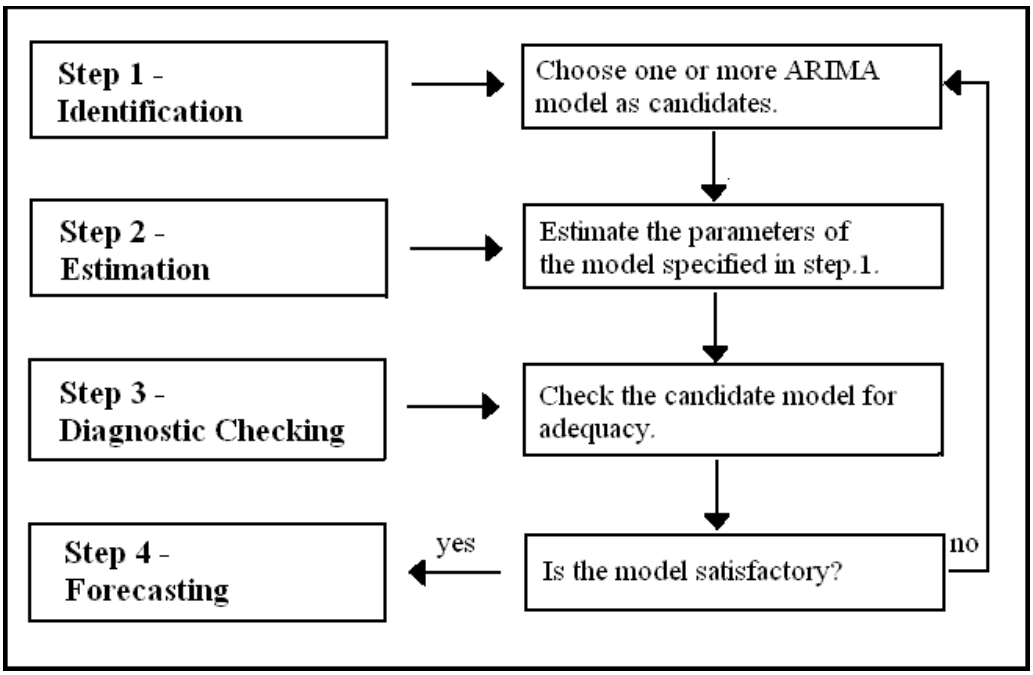

Figure 1 Steps of the Box-Jenkins Approach

3.1 Unit Root Test

Many economic and financial time series exhibit trending behavior or nonstationarity in the mean. In the presence of nonstationary variables, there might be what Granger and Newbold (1974) call a spurious relationship. In a spurious regression, there are a high coefficient of determination (R^2) and *t*-statistic that appear to be significant, but the results are without any economic meaning. Unit root tests can be applied to determine if trending data should be first differenced or regressed on deterministic functions of time to render the data stationary. Moreover, economic and financial theory often suggests the existence of long-run equilibrium relationships among nonstationary time series variables. If these variables are I (1), then co-integration techniques can be used to model these long-run relations. Hence, pre-testing for unit roots is often a first step in time series model.

3.2 Chow Breakpoint Test

Estimation of all the parameters (Φ_i, r_j, d, p_j) of the SETAR model at the same time is difficult. One way to determine threshold model order (lag length) is to set up an initial value and then compare it to the others. Hansen (1996) presents a general framework for testing the null of linearity against the alternative of threshold auto-regression. That delivers valid inference when the threshold value r_j and delay d are unknown a priori, in the sense that they have to be learnt from the data. Later, Hansen (2001) quoted Chow test in finding structural change of unknown timing. Researchers have only two choices: to pick an arbitrary candidate breakpoint or to

pick a point based on some known feature of the data. In the first case, the Chow test may be uninformative, as the true breakpoint can be missed. In the second case, the Chow test can be misleading, as the candidate breakpoint is endogenous. Since the results can be highly sensitive to these arbitrary choices, a sound scientific practice method will be better.

3.3 SETAR Model

Threshold autoregressive (TAR) models are one class of non-linear autoregressive models. Such models are a relatively simple relaxation of standard linear autoregressive models that allow a locally linear approximation over a number of states. According to Tong (1990, p.99), the threshold principle 'allows the analysis of a complex stochastic system by decomposing it into a set of smaller sub-systems'. The TAR model assumes that the regime is determined by the value of a threshold value. Referring to Brooks (2002, p.560), a simple TAR model is given by:

$$\mathrm{TAIEX}_t = \begin{cases} \mu_1 + \Phi_1 \mathrm{TAIEX}_{t-1} + u_{1t} & \text{if } \mathrm{TAIEX}_{t-d} < r \\ \mu_2 + \Phi_2 \mathrm{TAIEX}_{t-1} + u_{2t} & \text{if } \mathrm{TAIEX}_{t-d} \geq r \end{cases} \tag{1}$$

The dependent variable TAIEX_t is the Taiwan Stock Exchange Capitalization Weighted Stock Index, it is purported to follow an autoregressive process with intercept coefficient μ_1 and autoregressive coefficient Φ_1 if the value of the state-determining variable lagged d periods, denoted TAIEX_{t-d} is lower than the threshold value r. If the value of the state-determining variable lagged d periods, is equal to or greater than that threshold value r, TAIEX_t is specified to follow a different autoregressive process, with intercept coefficient μ_2 and autoregressive coefficient Φ_2. But what is TAIEX_{t-d}, the state-determining variable? It can be any variable that is thought to make TAIEX_t shift from one set of behavior to another. Obviously, financial or economic theory should have an important role to play in making this decision. If $d=0$, it is the current value of the state-determining variable that influences the regime that TAIEX is in at time t, but in many applications d is set to 1, so that the immediately preceding value of s is the one that determines the current value of y.

3.4 Out-of-sample Forecasting Performance

Briefly introduce, one-setp-ahead forcasting is expects the value of t-1's day standing on the t's day. This evaluation technique was employed in this paper to compare the relative forecast performances of the two models. We cut sample into two parts: in-sample and out-of-sample. The first 90% of data is in-sample and the rest of 10% is out-of-sample. After re-estimating a new model by using the in-sample data,

we use the new model to forecast the fitted value. Then we can compare the forecasted value with the original data.

The differences between the fitted value and the original value are errors. We adopted more than one criterion for valid robustness. In practice, forecasts would usually be produced for the whole of the out-of-sample period, which would then be compared with the actual values, and the difference between them aggregated in some way. (Brooks, 2002)

Those four criterion were Mean Squared Error (MSE), Mean Absolute Error (MAE), Adjusted Mean Absolute Property Error (AMAPE), and Mean Absolute Property Error (MAPE) respectively. MSE provides a quadratic loss function, and so may be particularly useful in situations where large forecast errors are disproportionately more serious than smaller ones. This may, however, also be viewed as a disadvantage if large errors are not disproportionately more serious, although the same critique could also, of course, be applied to the whole least squares methodology. Indeed Dielman (1986) goes as far as to say that when there are outliers present, least absolute values should be used to determine model parameters rather than least squares. That's why we adopted both of the MSE and MAE. Makridakis (1993) argues that MAPE is a relative measure that incorporates the best characteristics among the various accuracy criteria. Therefore, we took MAPE in consideration too. Besides the upper mentioned criteria, we also took AMAPE for completeness.

4 Empirical Results

4.1 Sample Description

The daily data of the Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) covering the periods from June 3, 2005 to December 31, 2009 for a total of 1,242 observations are plotted on Figure 2. The dates are on x-axis. The indices are on the y-axis. There are two important values regarding the data to be made. Those are 9809.88, the peak on October 29, 2007 and 4089.93, the bottom on November 20, 2008. Affected by the Global Financial Crisis, there are only 13 months between the high and low in Taiwan stock market. In comparison, TAIEX has been steadily growing for almost three years since 2005, the index increased from 6143.12 (January 3, 2005) to 9809.88 (October 29,2007). Suddenly, a downturn happened in the end of 2007. TAIEX dropped to 7408.4 in January 23, 2008, and had a rebound to 9197.41 (May 16, 2008). Many people believe that they finally get through the financial crisis but it just the opening. The index keeps falling sharply till the bottom 4089.93 (November 20, 2008).

During such a terrific crisis, we think the structural change might happen, especial in the downturn period. We doubt it because the indices have fallen sharply during the period, and hit the historical low.

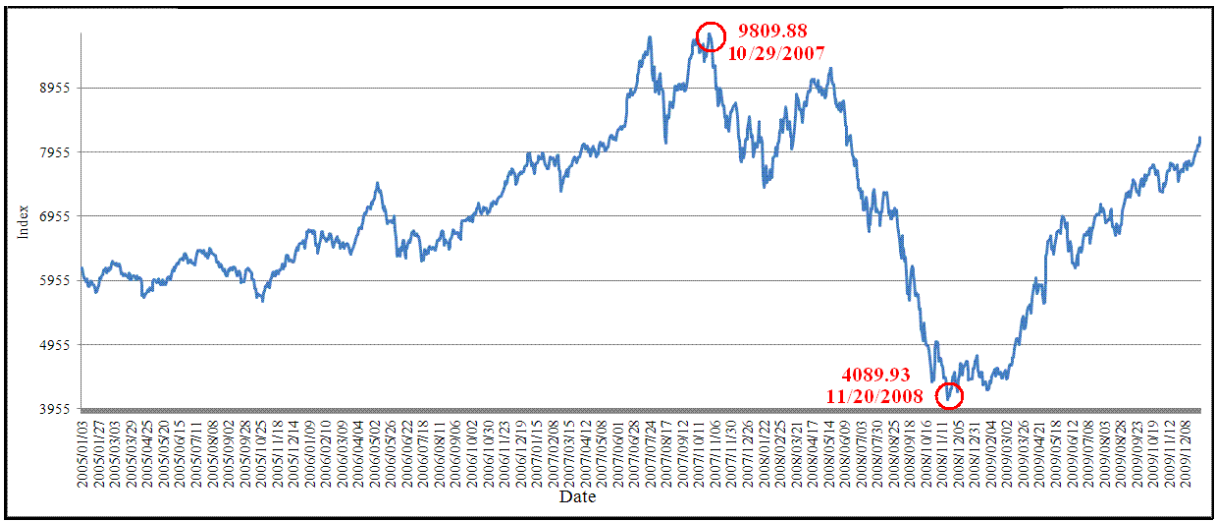

Figure 2 Trend of Taiwan Stock Exchange Capitalization Weighted Stock Index

Note: Horizontal axis represents date, vertical axis is the index.

Sample description of the Taiwan Stock Exchange Capitalization Weighted Stock Index is in Table 1. The mean is 6983.75 and the median is 6865.15. Usually, if the mean and median are close, it means that the data are symmetric around the mean. For every bit over the mean on one side, there's a corresponding bit under the mean on the other side, balancing it out. Maximum is 9809.88 and the minimum is 4089.93. Standard deviation is 1258.611 and the coefficient of variation is 18.02%. The coefficient of variation represents the ratio of the standard deviation to the mean, and it is a useful statistic for comparing the degree of variation from one data series to another, even if the means are drastically different from each other. We have one sample here, but still we can see the degree of variation.

By the bar chart of sample description, we can see the sample that the majority of series are located in the index between 6000 and 7000. Then slowly decrease over 7500. A small number of index clusters around 4500. And only a little few series located nearly 5500. This means how fast the index goes down.

Table 1 Sample Description

Sample	Mean	Median	Maximum	Minimum	Std. Dev.	CV (%)	Kurtosis
1242	6983.75	6865.15	9809.88	4089.93	1258.61	18.02	2.6032

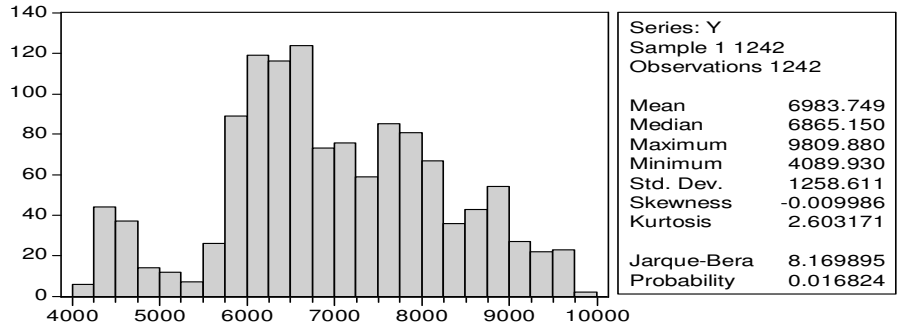

Figure 3 Sample Description

4.2 Results of Unit Root Test

This step is to check the data stability. We test unit root. The p-values of the test statistic were bigger than the 10% significance level through the three models above. Therefore, the null hypothesis (H_0), in which there is a unit root, was rejected. After the series was taken the first difference, all the p-values of the statistics were smaller than 1% significance level.

4.3 Results of ARIMA Model Selection

In order to identify the appropriate model parameter of ARIMA (p, d, q), we adopt the Akaike's information criterion (AIC). From the unit root test, we obtain d to be 1. As to parameters p and q, we run the regression through the combinations of from p =1 to p=10 and from q=1 to q=10. For sake of saving space, we just list the top 10 with higher AIC values here. As shown in Table 2, the model with the smallest value (9.14334) of AIC is the optimal ARIMA (6, 1, 9) chosen.

Table 2 Results of Model Selection of ARIMA

Akaike's information criterion (AIC)		
Rank	Models	AIC
1	ARIMA (6, 1, 9)	9.14340
2	ARIMA (9, 1, 7)	9.14403
3	ARIMA (9, 1, 9)	9.14545
4	ARIMA (9, 1, 6)	9.14505
5	ARIMA (8, 1, 9)	9.14871
6	ARIMA(10, 1, 9)	9.14879
7	ARIMA (6, 1, 6)	9.14978
8	ARIMA (9, 1, 8)	9.15125
9	ARIMA (6, 1, 5)	9.15178
10	ARIMA(6, 1, 10)	9.15368

Note: Models in the Table are ranked by the AIC.

4.4 Results of Chow Breakpoint Test

In many applications, it is useful to estimate as the structural change occurred. Treat the date of structural change as an unknown parameter. Either visual observation

or model estimation is an appropriate way. In the present study, we adopt the model estimation. An obvious candidate for a breakpoint estimate is the date that yields the largest value of the Chow test sequence. It turns out that this is known to be a good estimate only in one special case-in a linear regression when the Chow test is constructed with the homoskedastic form of covariance matrix.

In the present study, we employ Chow breakpoint test to find the breakpoint. According to Hansen (2001), we run Chow testing starting from 5% of the sample set, and then we plot the F-statistic. In Figure 3, it seems that the breakpoint is 680_{th} sample point with F-statistic (5.150791), in which the breakpoint is 7811.8 on June 26, 2008. We mark it on the stock index trend as in Figure 4.

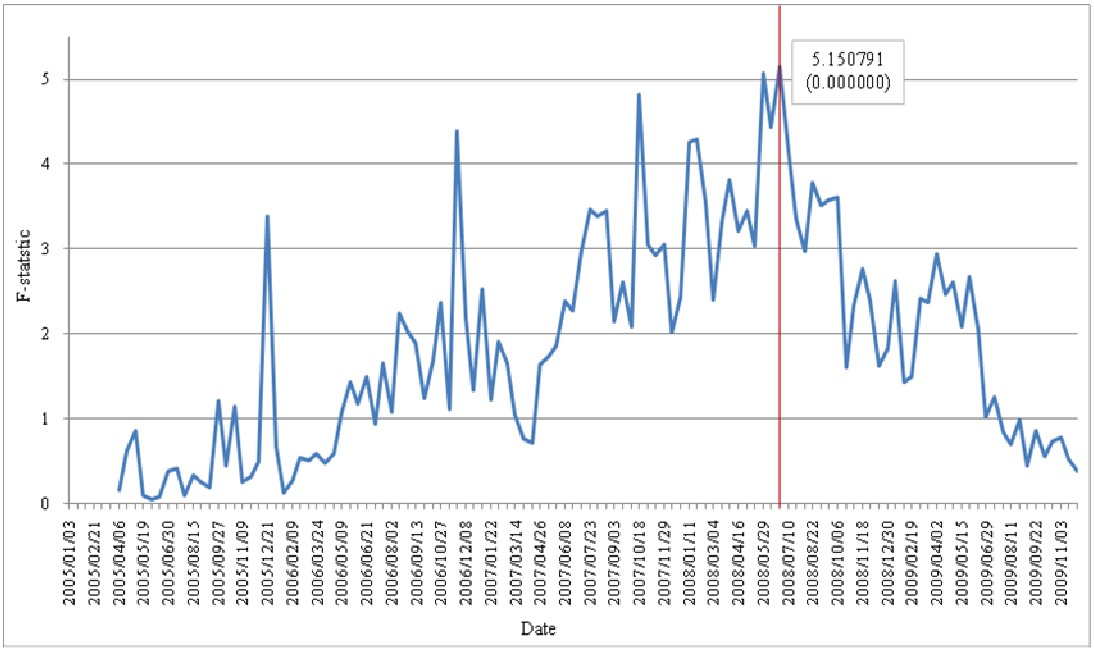

Figure 4 Breakpoint by Using Chow Test

It is interesting since the breakpoint just falls right in the quarter of the falling period. It just cut the trend into two different parts. The former regime looks more regular than the later regime does. The index goes up steady in the former regime but sharply decline after the breakpoint. This just confirms our earlier words. Been through a sudden volatility, the index have a structural change. The trend is completely different afterward.

The trend seems to have some technically analytic meaning. However, this is another contribution of this paper. We leave this interesting finding for the following research since this paper is a time series demonstrative study.

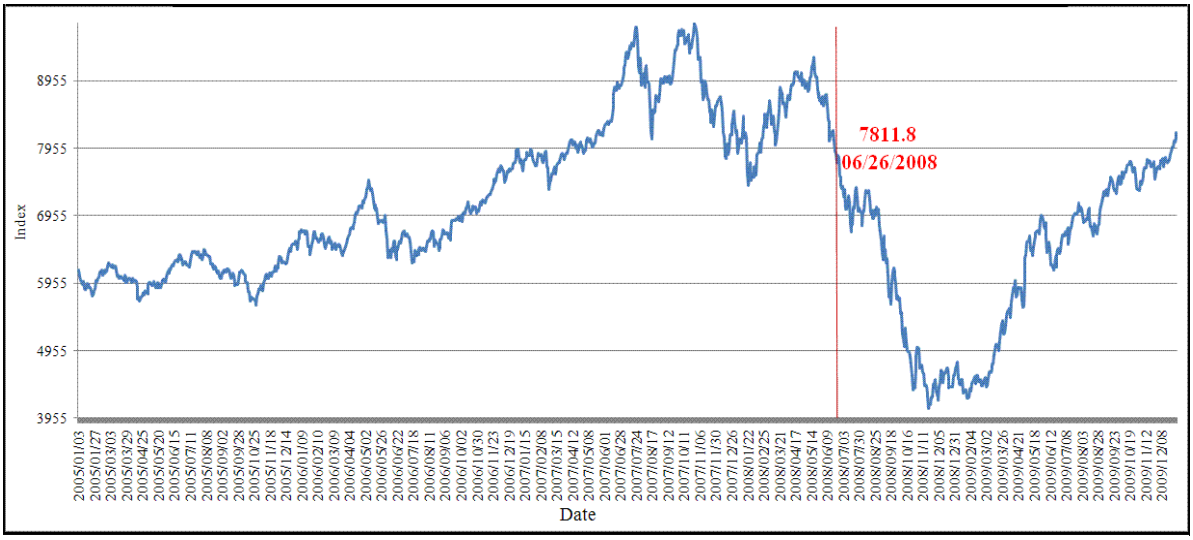

Figure 5 Breakpoint in Taiwan Stock Exchange Capitalization Weighted Stock Index

Note: The breakpoint shown in date is June 26, 2008.

4.5 Results of SETAR Model

In our self-exciting threshold autoregressive (SETAR) model, we assume that a variable $TAIEX_t$ is a linear autoregression within a regime. As there are two regimes in our research period, the model could be written as SETAR (2, p, p)

There is a structural change on the date of June 26, 2008. Shown in Table 3, Panel A is regime 1 of the two-regime SETAR and panel B is regime 2. We select the most appropriate model by minimizing value of AIC. We build up a two-regime SETAR model, SETAR (2, 6, 4).

Table 3 Results of Model Selection of SETAR

Panel A: Regime 1 (Included observations: 860)		
Rank	Models	AIC
1	AR(6)	11.87146
2	AR(3)	11.87302
3	AR(7)	11.87337
4	AR(4)	11.87550
5	AR(8)	11.87583
6	AR(5)	11.87588
7	AR(9)	11.87906
8	AR(10)	11.88004
9	AR(1)	11.88055
10	AR(2)	11.88221
Panel B: Regime 2 (Included observations: 382)		
Rank	Models	AIC
1	AR(4)	11.06017
2	AR(1)	11.06027
3	AR(3)	11.06395
4	AR(2)	11.06402
5	AR(5)	11.06488
6	AR(8)	11.07564
7	AR(7)	11.06884
8	AR(6)	11.07065
9	AR(10)	11.07328
10	AR(9)	11.07362

Note: Models in the Table are divided into two regimes and ranked by the AIC.

4.6 Results Out-of-sample Forecasting Performance

In this section, we check out-of-sample forecast performance of the two models. By checking multi-criteria (MSE, MAE, AMPE, and MAPE) on ARIMA and SETAR, we can compare the residual of these two models. For the mean squared error (MSE), SETAR is smaller than ARIMA with 4479.28 and 5744.65. This means SETAR has a fewer errors in the standard of MSE. For the mean absolute error (MAE), the adjusted mean absolute property error (AMAPE) and mean absolute property error (MAPE), SETAR is also smaller than ARIMA with fewer errors. According to our result shown in the Table 4, the SETAR model is better than the ARIMA model over the sample period (second regime).

Table 4 Comparison of Forecasting Power

	MSE	MAE	AMAPE(%)	MAPE(%)
SETAR	4479.28	45.51	90.68	89.02
ARIMA	5744.65	58.32	304.20	190.91

Note:

1. SETAR model is SETAR (2, 6, 4) and ARIMA model is ARIMA (6, 1, 9).
2. Mean Squared Error

$$MSE = \frac{s}{T-(T_1-1)}\sum_{t=T_1}^{T}(y_{t+s} - f_{t,s})^2 \quad (2)$$

Where T is the total sample size (in-sample + out-of-sample), and T_1 is the first out-of-sample forecast observation. $f_{t,s}$ denoted s-step-ahead forecasts of a variable made at time t. And in this paper, we simply use one-step forecast.

3. Mean Absolute Error

$$MAE = \frac{1}{T-(T_1-1)}\sum_{t=T_1}^{T}|y_{t+s} - f_{t,s}|^2 \quad (3)$$

4. Adjusted Mean Absolute Property Error

$$AMAPE = \frac{100}{T-(T_1-1)}\sum_{t=T_1}^{T}\left|\frac{y_{t+s} - f_{t,s}}{y_{t+s} + f_{t,s}}\right| \quad (4)$$

5. Mean Absolute Property Error

$$MAPE = \frac{100}{T-(T_1-1)}\sum_{t=T_1}^{T}\left|\frac{y_{t+s} - f_{t,s}}{y_{t+s}}\right| \quad (5)$$

In Figure 6, it shows the one-step-ahead forecasted value from both models compared with the original stock index data. Noticeably, the SETAR forecasting series is much closer to the original stock index data than those of ARIMA model do. Since ARIMA have no consideration in structural change, the estimated errors will be larger than SETAR. The difference between SETAR and the original series are within 5, and the difference between ARIMA and the original series are within 35. Check out the fitted value in page number 25 and 26. We can see that the forecasts of the SETAR model are always closer to our original data than that of the ARIMA. Therefore, it may suggest that the SETAR have a stronger forecasting power than ARIMA.

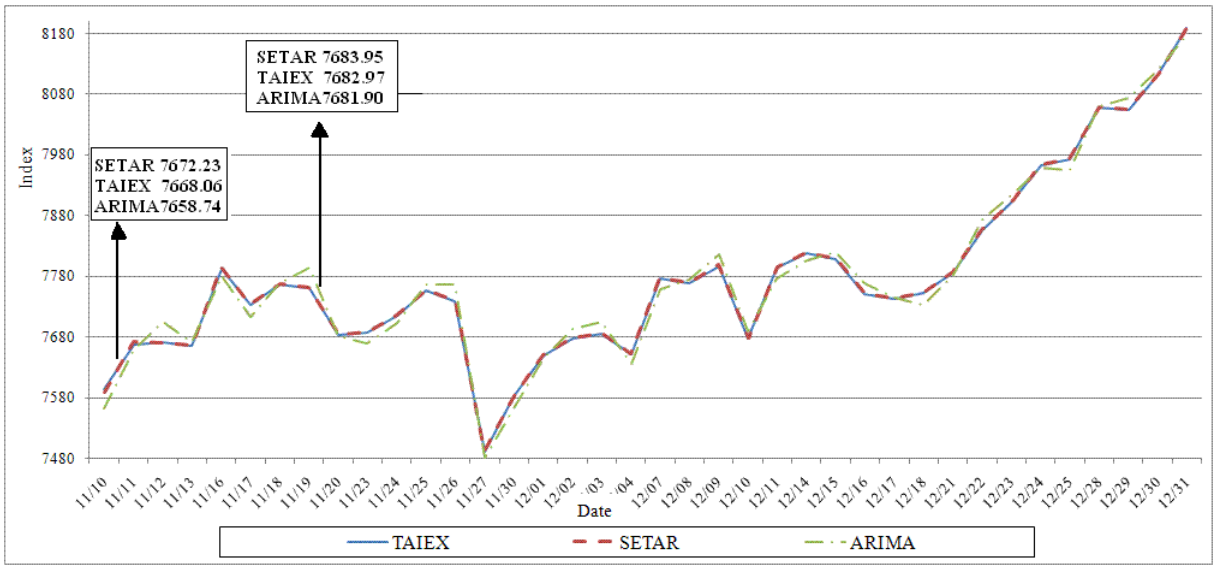

Figure 6 Original Series and Predicted Value from the SETAR and ARIMA Model using Forecasting Method (2009)

Note:

1. TAIEX is the trend of Taiwan Stock Exchange Capitalization Weighted Stock Index.
2. SETAR is the trend of fitted value estimated by SETAR (2, 6, 4).
3. ARIMA is the trend of fitted value estimated by ARIMA (6, 1, 9).

Figure 7 shows a 95% forecasting interval of the ARIMA model by using the one-step-ahead forecasting method and also the locations of the forecasted values for both models using the same forecasting method. It is easy to see that the out-of-sample one-step-ahead forecasting values of the SETAR model set themselves within the one-step-ahead 95% confidence forecasting intervals of ARIMA model respectively. This indicates that the 95% confidence forecasting intervals of both models overlap each other to some extent. Taking their standard deviation into consideration, we could conclude that the forecasting intervals of the two models are not different from each other. As a result, the one-step-ahead forecasting performances of both models are significantly different from each other.

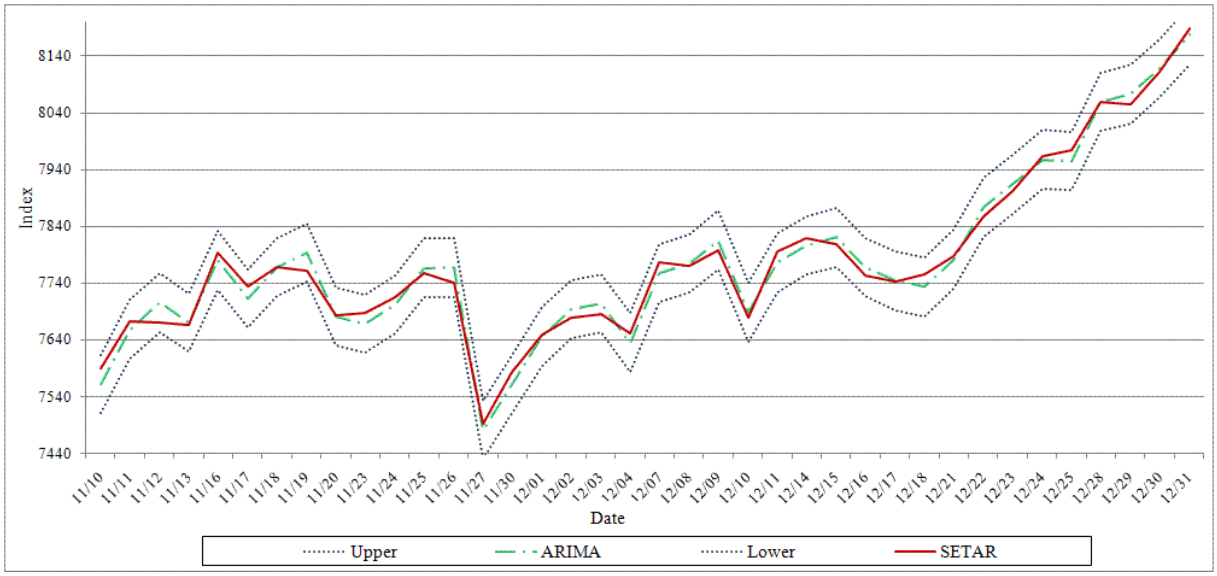

Figure 7 Forecasting Interval of the ARIMA Model and the Forecasted Value from Both Models (2009)

Note:
1. Upper is the upper value of ARIMA (6, 1, 9) interval.
2. ARIMA is the trend of fitted value estimated by ARIMA (6, 1, 9).
3. Lower is the lower value of ARIMA (6, 1, 9) interval.
4. SETAR is the trend of fitted value estimated by SETAR (2, 6, 4).

5. Conclusions

At first, we believed there is no way for a linear regression to suit a series forever where stock prices follow a non-linear trend. Due to the economic environment changing, the stock market will be affected and change over time. Therefore, non-linear regression should be better than linear regression in the stock market.

First step of handling time series data is to check stationary state in the mean. We found out there was a unit root existed so we analyzed the first-difference. Next, we constructed an ARIMA model by using AIC selection criteria. And we build up a SETAR model by Chow breakpoint test and AIC as well. The step used Chow breakpoint test to find the breakpoint was accorded to Hansen's (2001) published research.

After the breakpoint test, we got a breakpoint at 7811.8 which happened on June 26, 2008. That is right in the middle of the financial crisis which means the crisis did affect the data generating process. Afterward, we checked the forecasting power by four criteria (MSE, MAE, AMAPE, and MAPE) and all of those standards showed that SETAR has a stronger predicting power than ARIMA. The results also support previous assumptions of this thesis. Non-linear SETAR model is better than linear ARIMA model in Taiwan Stock Exchange Capitalization Weighted Stock Index from June 3, 2005 to December 31, 2009.

There are a number of potential explanations for the findings in this paper, suggesting directions for further research. One is the technical meaning of the break

date, another is the need for better forecast evaluation techniques.

References

1. Boero, G. and Marrocu, E. (2002), The performance of non-linear exchange rate models: a forecasting comparison, *Journal of Forecasting*, Vol. 21(7), 513-542.
2. Boero, G. and Marrocu, E. (2003), *The performance of SETAR models: a regime conditional evaluation of point, interval and density forecasts. Working Paper.*
3. Box, G. and Jenkins, G. (1976), Time Series Analysis: Forecasting and Control, *San Francisco: Holden-Day.*
4. Brooks, C. (2002), *Introductory Econometrics for Finance*, Cambridge: Cambridge University Press.
5. Clements, M. and Smith, J. (2001), Evaluating forecasts from SETAR models of exchange rates, *Journal of International Money and Finance*, Vol. 20(1), 133-148.
6. De Gooijer, J., Ray, B. and Kräger, H. (1998), Forecasting exchange rates using TSMARS, *Journal of International Money and Finance,* Vol. 17(3), 513- 534.
7. Dielman, T. (1986), A comparison of forecasts from least absolute value and least squares regression, *Journal of Forecasting,* Vol. 5(3), 189-195.
8. Granger, J. and Newbold, P. (1974), Spurious regressions in econometrics, *Journal of Econometrics,* Vol. 2(2), 111-120.
9. Hansen, B. (1996), Inference when a nuisance parameter is not identified under the null hypothesis, *Econometrica*, Vol. 64(2), 413-430.
10. Hansen, B. (2001), The new econometrics of structural change: dating breaks in U.S. labor productivity, *Journal of Economic Perspectives*, Vol. 15(4), 117-128.
11. Kapetanios, G. and Shin, Y. (2006), Unit root tests in three-regime, *Econometrics Journal*, Vol. 9(2), 252-278.
12. Kräger, H. and Kugler, P. (1993), Nonlinearities in foreign exchange markets: a different perspective, *Journal of International Money and Finance*, Vol. 12(2), 195-208.
13. LeBaron, B. (1992), LeBaron, Some relationships between volatility and serial correlations in stock market returns, *Journal of Business* Vol. 65(2), 199-219.
14. Makridakis, S. (1993), Accuracy measures: theoretical and practical concerns, *International Journal of Forecasting*, Vol. 9(4), 527-529.
15. Tong, H. (1977), Some comments on the Canadian lynx data (with discussion). *Journal of the Royal Statistical Society. Series A (General),* Vol. 140(4), 432-436.
16. Tong, H. (1978), On a threshold model, in Chen, C. H.(ed.), Pattern Recognition and Signal Processing*, Amsterdam: Sijhoff and Noordoff*.
17. Tong, H. and Lim, K. S. (1980), Threshold autoregression, limit cycles and cyclical data (with discussion), *Journal of the Royal Statistical Society*, Series B, Vol. 42(3), 245-292.
18. Tong, H. (1990), *Non-Linear Time Series. A Dynamical System Approach*, Oxford, UK: Oxford University Press.
19. Tsay, R. (1989), Testing and modeling threshold autoregressive processes, *Journal of the American Statistical Association*, Vol. 84(405), 231-240.