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Macroeconomic Risk, Long-Run Risk and Asset Pricing Puzzles

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Abstract

Several studies have suggested that time-variation in cash-flow risk and expected returns are important issues in long-horizon asset pricing. In other words, systematic or macroeconomic sources of risky factors are not constant, and they will vary across different investment horizons.

However, conditional asset pricing theory is not an elixir to support the empirical results from time series data. On the contrary, more and more financial economists are in dispute even though conditional valuation models including the time-varying risky factors can have explained some asset pricing anomalies such as momentum, high value premium, and decreasing equity premium in the long run.

In this paper, we will introduce log-linear asset pricing framework of discounted cash flow (a present-value model) and log-linear consumption-based asset pricing model to explain an important conclusion of the declining equity premium for long horizons. Moreover, we have divided market beta into cash-flow beta and discount-factor beta to discuss what role macroeconomic risk and duration risk play in different investment horizons.

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1. Introduction

Several studies have suggested that time-variation in cash-flow risk and expected returns are important issues in long-horizon asset pricing. In other words, systematic or macroeconomic sources of risky factors are not constant, and they will vary across different investment horizons. Those time-varying risks in the long-run seem to improve the explanatory power of unconditional asset pricing models, such as CAPM or consumption-based CAPM (CCAPM).

Conditional time-variation asset pricing model indicates that rational expected investors update their information set period by period to determine the stock prices according to market beta and expected cash flow. Those models highlight that the trade-off relationships among expected returns and risks for assets should shift over time in predictable ways, and these shifts tend to persist over long periods of time.

However, conditional asset pricing theory is not an elixir to support empirical results from time series data. In contrast, more and more financial economists are in dispute even though the conditional valuation models including the time-varying risky factors could have explained some asset pricing anomalies (Lewellen and Nagel, 2006).

One of those anomalies is the *momentum* suggested by Jegadeesh and Titman (1993) and it describes that stocks with high returns in the previous period should continue to outperform those with low prior returns. Another two are defined by Fama and French (1993). In them, the *small-size premium* (SMB) denotes that small stocks would outperform large stocks, and the *value premium* shows that firms with high book-to-market (B/M) ratios would outperform those with low B/M ratios.

It is likely that investors may have opportunities to earn higher returns as they construct portfolios with properties of those anomalies because the prices of those portfolios would temporally deviate from the fundamental values. Nonetheless, those anomalies are not only inconsistent with asset pricing models but also have violated the hypothesis of market efficiency.

Moreover, what many financial authors care is whether those abnormal returns could be continuous in the long-run and if those anomaly returns can be explained or predicted by existing asset pricing theory. In fact, those excessive returns are likely to decline in the long run based on the significant results (Franzoni, 2006; Lettau et al., 2008; Lettau and Watchter, 2007).

In this paper, we will introduce log-linear asset pricing framework of discounted cash flow (a present-value model) and log-linear consumption-based asset pricing model to explain an important conclusion of the declining equity premium for long horizons. Moreover, we have divided market beta into *cash-flow beta* and *discount-factor beta* in order to discuss what role macroeconomic risk and duration risk play in different investment horizons.

2. Asset pricing puzzles and distinct sources of risks

With the assumption of time-varying rational and conditional expectation in asset pricing model like CCAM, a representative agent or social planner is given the incentive to predict the equity premium to determine his intertemporal consumption and investment decision, and maximize his whole life utility of consumption. Of course, the abnormal equity premium derived from the anomalies are of the predicted condition from the agent's time-varying information sets and are restricted to dynamic budget and rational bubble constraints (Campbell, 2003).

Whether if one uses the discount cash flow model (DCF) to evaluate the stock price by weighted average cost of capital forecast with conditional CAPM or CCAPM to find out the equity premium period by period, macroeconomic factors or systematic risks would play center roles, while systematic risks have signaled for the most of information about cash flow and investor's risk preference in future economic condition. Besides, systematic risks are also independent of interest, and they have summarized some of the relevant characteristics of the firm's fundamentals.

As a whole, asset pricing model exhibits that if investor is to put up with the systematic risk, he must be compensated with some units of the price risk denoted as risk premium. This idea holds in both of the conditional CAPM and CCAPM. For every occasion, since one price of risk is used to determine asset prices, the price of risk would be the same for all assets. In addition, the concept of transformation between CAPM and CCAPM should be mentioned as both express that risk premium is measured in quantity of risk multiplied by price of risk plus risk-free rate.

However, difference remains with quantity of risk between them, and the quantity of risk in CAPM denoted as market beta shows the covariance of individual asset's excess return and risk price, moreover, the quantity of risk in CCAPM defined by consumption beta shows the co-variation between individual asset's excess return and the return of aggregate consumption portfolio. Furthermore, CCAPM also suggests that the path of aggregate consumption may have affected stochastic discount factor (SDF).

Financial economists are interested with the quantity of risk varying with time. Most of them have used past data to estimate market beta via window regressions in conditional

CAPM (Lewellen and Nagel, 2006), but it is difficult to find out consumption beta as consumption growth is hard to forecast in the long run (Campbell, 2003).

Conditional asset pricing model is also viewed as the prediction in stock return or the forecast of risk premium time by time. If the state variables (macroeconomic factors) could predict the improvement in economy, stocks' betas would become smaller. Then, some variation in systematic economy such as aggregate consumption risk has been eliminated due to efficient foresight, while investor would demand low rate of return folding risky assets.

Empirically, equity premium especially for value premium and small-size premium have decreased in the long run. Based on the viewpoint of CCAPM, Lettau and Watchter (2007) had suggested that value premium is high as correlated to macroeconomic factors, with dividends of value stocks paid out recently in contrast to growth stock dividend paid far in the future so that value stock has higher macroeconomic risk than that of growth stock in the short term, but lower duration risk of cash flow in the long run.

Duration is a powerful instrument for investor to know from how many periods of time he would recover his cost of investment. Investor would have suffered larger duration risk if he needs to spend longer time to regain the stream of cash flows from the asset. Besides, larger macroeconomic risk of value stocks in the short-run implies that value firms would co-vary more with aggregate dividend sequences, but growth firms would co-vary more with discount rate (stochastic discount factor, SDF).

On the other hand, value stocks of lesser duration stocks can provide a more substantial evidence to explain the declining equity premium of the long-lasting assets.

The decreasing equity premium in the long run also is verified of market betas dropping progressively, and Franzoni (2006) discovered that market betas of value and small stocks have decreased about 75% in the second half of the twentieth century. When market beta is divided into cash flow and expected return news components exactly as suggested by Campbell and Volteenaho (2004), they have confirmed that the payoffs of those stocks are less sensitive to business conditions in the long run.

The finding of Franzoni (2006) seems to support that conditional CAPM can explain as much as 80% of the value premium if investors could have tied their expectations of risk to these stocks and the high values of beta prevailing in the early years. However, this assumption would not necessarily correspond to fact.

Lettau et al. (2008) has tried to explain the declining equity premium with macroeconomic risk as their research extends previous studies of long-horizon co-integration constrain among aggregate consumption, aggregate wealth, and human capital in the vector autoregressive model to estimate expected risk premium (Lettau and van Nieuwerburgh, 2001). Moreover, the aggregate dividend could have replaced the aggregate wealth due to the unobservable process of aggregate wealth (Lettau and van Nieuwerburgh, 2005).

Those papers have exhibited distinct components of systematic risk playing different roles to evaluate the price of assets, and one of them is *cash-flow beta* that reveals news about cash flow, and the other is *discount-factor beta* that reflects news about expected return.

Cash-flow beta arises if the streams of asset dividends are highly correlated to macroeconomic factors or business conditions. For assets with high covariance of dividend growth and aggregate consumption growth, their equity premium will increase as those assets cannot help investors to diversify their contemporary consumptions risks. Therefore, whether the recession or boom, the dividends of those pro-cyclical assets will co-vary with aggregate consumption so as to enhance the variation in investors' consumptions, while cash-flow beta will be of bad beta against consumption smoothing.

In contrast, discount-factor beta will, similar to duration risk, decline in investment horizon, and it reduces equity premium over time.

Furthermore, with a comprehensive look at the empirical results, we can conclude that if macroeconomic state variables could have predicted an improvement with the economy as well as the conditional asset pricing models we could find those betas of value and small stocks in decrease. Such result would not only apply to cyclical movements of beta, but also to decrease in the long run as some of these variables display trending behavior reckoned to be related to long-term improvement with business conditions.

And the declining equity premium induces other substantial issues about stock return predictability at long horizons, and if it holds, which macroeconomic state variables could predict the equity premium in the pricing model efficiently?

As of intuition, those macroeconomic state variables can be selected from the works of asset pricing, and there are many authors such as Campbell and Shiller (1988a and b), Fama and French (1998), Stambaugh (1999), Goyal and Welch (2003, 2004), Lewellen (2003), Campbell and Yogo (2006) and Ang and Bekaert (2007) who have used dividend yield or price-dividend ratio derived from discount cash flow model to examine the predictive power on excess stock returns.

Most of them found that those financial ratios with the ability to estimate excess returns can be best visible at short horizons with short rate as an additional independent variable. Interestingly, at short horizons, the short rate can strongly and negatively predict excess returns, while the predictive power of the financial ratios is weakened at long horizons.

Ang and Bekaert (2007) had also discovered the strong role of earnings yield as a predictive instrument, not for excess returns, but for future cash flows.

These results are consistent with the declining equity premium since the predictability is evidenced in a present value model showing that the variation in the discount rates still dominates the dividends for long horizons (Ang and Bekaert, 2007). Such phenomenon also implies that the deviation of asset price should temporally converge to long-run fundamental value due to declining duration of risky cash flows as well as better predictable instruments about microeconomic risks in the long run.

To appreciate how the declining duration of risky cash flows affects equilibrium asset prices, it is helpful to study how risk premium differs with relative exposure to dividend growth in the long-run versus short-run consumption risk differs (Yogo, 2006; Lettau and Watchter, 2007). And it is an important concept to link up asset prices with consumption risks from CCAPM.

Also, the price of asset will be lowered if its payoff co-varies positively with consumption; conversely, the price of asset's price will be raised if it co-varies negatively with consumption. This is mainly due to the fact that investors do not like uncertainty about consumption (Cochrane, 2005).

If investor buys an asset whose payoff co-varies positively with consumption – meaning that it pays off well when investor is already feeling wealthy, and vice versa, such asset will turn the seam of investor's consumption more volatile (bad asset). Thus, low price is needed to induce the investor him to buy such an asset. If investor buys an asset whose payoff co-varies negatively with consumption, it helps to smooth consumption and so it is more valuable than its expected payoff (good asset).

Yogo (2006) has applied this notion to explain *value premium.* Value stocks are in greater demand more than equity premium because their cash flows covary positively with aggregate consumption than those of growth stocks, and it implies that value stocks have high *cash-flow betas* or short-run consumption risks.

However, the consumption risks for value stocks will decrease in time as the duration risks of value stocks will shrink to nothing for the long horizons, and the cash flows of value stocks will, never again, work upon the consumption risks.

Another reason in response to the declining equity premium is the improvement of stock returns' prediction. And financial economists are devoted to finding out some credible macroeconomic factors to explain the declining equity premium, and they have also developed some efficient econometric approaches to examine the return predictability of stock (Campbell and Yogo, 2006).

To study the long-run relationship between declining consumption risks and the equity premium, many authors have begun with the consumption-based asset pricing model, and found out that some aggregate macroeconomic factors such as aggregate consumption, aggregate dividends, or labor income could be the candidates to forecast long-term stock returns (Lettau and Nieuwerburgh, 2001, 2005; Santos and Veronesi, 2006).

An important issue to note is that aggregate housing consumption should not be a case of negligence because it will not only provide residential services but also a large proportion in the evolution of household's wealth. When the wealth ratio of housing changes, it may affect the stock-holding desire of household (Cocco, 2005) and change the equity premium (Piazzesi et al., 2007).

Lettau et al. (2008) has offered a very particular explanation to explain why equity premium decreases in the long run: a fall in macroeconomic risk, or the volatility of aggregate economy. And the idea can also be applied to stock return predictability. Empirically, a strong correlation is detected between low frequency movements in macroeconomic volatility and low frequency movements in stock market similar to the habit formation of consumption (Campbell and Cochrane, 2000) or variation in housing prices (Piazzesi et al., 2007).

3. Log-linear asset pricing frameworks and equity premium

In order to understand how macroeconomic risks and long-run risk involved in the equity premium, it is necessary for us to have a framework related movement in stock prices to movement with regard to expected future dividends and discount rates.

We should start from the present value model with time-varying expected stock return which suggests that stock pricing model is nonlinear, and this has forced researchers to make some simplifying assumptions available. The most common approach is to assume a conditional expected dividend growth following the discrete-state Markov process, while Markov structure of dividend growth makes its possible for us to solve present value model, but this assumption tends to be helpless solving complicate returns.

Another framework such as log-linear approximation suggested by Campbell and Shiller (1988a) can produce simpler form of solutions, and it is suitable for most of the overview of literature as it provides an accounting identity: Besides, high prices should be associated with low expected future returns or high expected future dividends. Similarly, high equity premium must be associated with upward revisions in expected future dividends or downward revisions in expected returns.

To observe such relationship, we apply log-liner approximation of the asset, and the log returns on the asset i, $r_{i,t+1} \equiv \log(P_{i,t+1}+D_{i,t+1}) - \log(P_{i,t})$ can be thus defined. We suppose that price is measured at the end of each period so that it would represent the claim to next period's dividend and price for consistency. Since log return is a non-linear function of log price and dividend, we use a first-order Taylor expansion approximate to log return around the average log dividend-price ratio, $(\overline{d_{i,t+1}-p_{i,t}})$. The approximation is

$$r_{i,t+1} \approx k + \rho p_{i,t+1} + (1-\rho) d_{i,t+1} - p_{i,t} \tag{1}$$

where $\rho \equiv 1 \big/ \left(1+\exp(\overline{d_{i,t+1}-p_{i,t}})\right)$ and $k \equiv -\log(\rho) - (1-\rho)\log(1/\rho - 1)$. Taylor approximation (1) would replace log of the sum of the stock price and the dividend in relation with the weighted average of the log stock price and log dividend. In empirical practice, log stock price would get a large weight, while log dividend a small weight because the dividend is, on average, much smaller than the stock price.

Obviously, Taylor approximation (1) is a first-order linear difference equation for log stock price at each time. Suppose that stock price should be non-explosive and restricted on a rational bubble constraint $\lim_{j\to\infty} \rho^j p_{i,t+j} = 0$, then solving forward, taking conditional expectations, and subtracting the current dividend on both side, we can get

$$p_{i,t} - d_{i,t} = \frac{k}{1-\rho} + E_t\left[\sum_{j=0}^{\infty} \rho^j \left(\Delta d_{i,t+1+j} - r_{i,t+1+j}\right)\right] \tag{2}$$

This equation indicates that log price-dividend ratio is high when dividends are expected to grow rapidly, or when stock returns are expected to be low. Intuitively, if the stock price is high today, there must either be high dividends or low stock returns in the future as viewed from the definition of the return and the terminal condition that stock price is non-explosive

So far, we have written asset prices as liner combination of expected future dividends and returns. Following after Campbell (2003), we can also write assets return as linear combinations of revisions in expected future dividends and returns. To substitute equation (2) by into equation (1), we can obtain

$$r_{i,t+1} - E_t\left[r_{i,t+1}\right] = \left(E_{t+1} - E_t\right)\sum_{j=0}^{\infty} \rho^j \Delta d_{i,t+1+j} - \left(E_{t+1} - E_t\right)\sum_{j=1}^{\infty} \rho^j r_{i,t+1+j} \tag{3}$$

where the notation $\left(E_{t+1}-E_t\right)$ indicates a revision in the conditional expectation between times t and t + 1. Besides, constant ρ will be derived from linearization process, and it can be interpreted as a discount factor.

And the equation notes that unexpected stock returns must be associated with changes in expectations of future dividends and real returns.

Moreover, this log-linear present value relationship allows one to express unexpected excess returns, or innovations of returns as news about dividends, news about real interest rates, and news about excess returns because the conditional asset pricing theory shows that expected return of risky asset is the linear combination of risk-free rate and risk premium (Campbell and Volteenaho, 2004). Then, asset i's unexpected excess return can be expressed as

$$\begin{aligned}\tilde{e}_{i,t+1} &\equiv r_{i,t+1}-E_t\left[r_{i,t+1}\right]=\left(E_{t+1}-E_t\right)\left\{\sum_{j=0}^{\infty}\rho^j\Delta d_{i,t+1+j}-\sum_{j=1}^{\infty}\rho^j r_{f,t+1+j}-\sum_{j=1}^{\infty}\rho^j r_{i,t+1+j}\right\} \\ &\equiv \tilde{e}_{d_{i,t+1}}-\tilde{e}_{r_{f,t+1}}-\tilde{e}_{r_{i,t+1}}\end{aligned} \quad (4)$$

The second equality in (4) introduces the simpler notation for dividend news $\tilde{e}_{di,t+1}$, real interest rate news $\tilde{e}_{rf,t+1}$ and excess return news $\tilde{e}_{ri,t+1}$.

Equation (4) is derived from asset pricing model, and it is rather the accounting identity than that of behavior formula. In it, we discover that the unexpected return varies with the revision of the conditional expectation to the present value of cash flow growth positively, but it varies with the revision of the conditional expectation to the present value of the discount rate negatively.

Since equity premium is only compensated by systematic risks, we denote a portfolio beta using unconditional variances and the covariance of portfolio and market innovations. Thus, beta is the unconditional covariance of the excess return innovation $\tilde{e}_i$ with the market innovation $\tilde{e}_m$, and it is divided by the unconditional variance of market innovation. The aspect of analysis is very important to help us to discuss the relationships among equity premium, macroeconomic risks, and long run risks.

$$\beta_{i,m}\equiv\frac{Cov\left(\tilde{e}_i,\tilde{e}_m\right)}{Var\left(\tilde{e}_m\right)} \quad (5)$$

Also, this beta is neither an unconditional beta (which would use returns themselves rather than innovations) nor a conditional beta (which would use conditional moments). However, it would coincide with a conditional beta if the conditional variance-covariance matrix of innovations has constant elements, or at least elements that have changed in proportion to one another.

The notation in (4) has the advantage that asset beta can be expressed as the sum of market betas of the three news components. From Equations (5), it follows directly that

$$\beta_{i,m} \equiv \frac{Cov(\tilde{e}_i,\tilde{e}_m)}{Var(\tilde{e}_{m,t+1})} = \frac{Cov(\tilde{e}_{d_{i,t+1}},\tilde{e}_{m,t+1})}{Var(\tilde{e}_{m,t+1})} - \frac{Cov(\tilde{e}_{r_{f,t+1}},\tilde{e}_{m,t+1})}{Var(\tilde{e}_{m,t+1})} - \frac{Cov(\tilde{e}_{r_{i,t+1}},\tilde{e}_{m,t+1})}{Var(\tilde{e}_{m,t+1})} \qquad (6)$$
$$\equiv \beta_{d_i,m} - \beta_{r_f,m} - \beta_{r_i,m}$$

where $\beta_{di,m}$ is the market beta of news about asset *i*'s cash flows, $\beta_{rf,m}$ the market beta of news about future real interest rates, and $\beta_{ri,m}$ the market beta of news about asset *i*'s future excess returns.

And it is convenient to observe that the quantity of risk for asset *i* arises if asset *i*'s unexpected cash flows are high as correlated to market innovation. On the other hand, the quantity of risk declines when the covariance of asset *i*'s unexpected returns and market innovation increases, or the covariance of unexpected real interest rates and market innovation become large.

The first term on the right hand side of Equation (6) is the *cash-flow beta*, the second the *term- structure beta*, and the third the *discount-factor beta.* Those betas can be applied to explain several asset pricing puzzles such as *small-size premium*, *value premium, momentum*, or declining equity premium.

For example, momentum traders would look to capture gains by riding 'winner' stocks and selling 'loser' ones. To participate in momentum investment, they will take a long position in an asset, which has shown upward trending price, or short sell a security that has shown downward trend.

The basic idea for momentum strategy is that once a trend is established it is more likely to continue in that direction than to move against the trend. Obviously, this strategy is profitable when *discount-factor beta* has less influence on the momentum portfolio. In other words, momentum traders believe that they can predict the change directions of portfolio price correctly without revising a large conditional expectation of excess return news.

In the case of value premium, most of the empirical works have placed their studies on the relative risk of value and growth stocks, and concluded that value betas tend to co-vary positively, and growth betas to co-vary negatively with the expected market risk premium (Ralitsa and Zhang, 2005; Yogo, 2006).

All of these studies have found that time-varying risk goes in the right orientation explaining value premium, but the inference of Ralitsa and Zhang (2005) is different from other studies because they have sorted betas on the expected market risk premium instead of the realized market excess return. Most interestingly, they have also found that this beta premium covariance is too small to explain the observed magnitude of the value premium within the conditional capital asset pricing model.

However, we have discovered some implications from these researches. First, the expected market beta cannot provide enough potential explanation for value premium, and the result is consistent with our setting of asset pricing model in which market beta is established with market innovation.

Second, the value premium is significantly positive as correlated to expected market beta or unexpected macroeconomic risks, and it suggests that high value premium can be created from large *cash-flow beta*. In contrast, the cash flows of growth stocks can generate far in the future, hence, the *cash-flow betas* of growth stocks are less than those of value stocks.

Third, stochastic discount factor is specified so that shocks to aggregate dividends are priced, but shocks to discount rate are not. These evidences imply that growth firms will co-vary more with the discount rate than that of value firms, which will co-vary more with cash flows.

As far as the declining equity premium aspects of analysis are concerned, we will offer value premium as the example. Lettau and Watchter (2007) had developed a duration-based model to explain why the risk premium of growth stock is less than that of value stock. In addition, they have suggested that value stocks have shorter recovery periods relative to growth stocks as value stocks are short-duration but high volatile assets.

On the other side, Bansal (2007) suggested that the declining equity premium on growth stock is due to less long-run risk on growth stock. Investor will prefer to resolve the uncertainty of his consumption only if the inter-temporal substitution of the consumption is larger than one. High inter-temporal substitution of the consumption implies more volatility in consumption path so that investor requires more excess return. Since the variation in the value stock's dividend is highly varying, value stock will lead investor to resolve his consumption risk early. This suggestion also coincides with the feature that shocks to aggregate dividends are priced in the stochastic discount factor.

Croce et al. (2007) had studied the role of information in asset pricing models with long-run cash flow risk. In order to illustrate the importance of information structure, they had showed how the implications of long-run risk paradigm regarding the properties of stock returns and cash flow duration are affected by information.

They have claimed that most of the investors are given with limited information as investors could observe the change in consumption and dividends in each period but they can not identify the consumption variation from the change in dividend. Therefore, investors will look on the variation in dividend as the consumption volatility and high variations in the dividend will exposure investors to a greater consumption risk and generate more risk premium.

The assumption of limited information conforms to our setting of fact, but there remain some difference between their model and ours. Our explanation is that investors, on one hand, prefer stationary stream of dividends, but they, on the other hand, want to earn more abnormal returns from heterogeneous belief or unexpected long run variations in business conditions. Hence, low *cash-flow beta* and high *discount-factor beta* may have reduced the equity premium for the long run.

Lettau and Watchter (2007) have offered similar suggestion, and they have reached the conclusion of their analysis that the decline occurs because of a fall in the dividend news beta.

Just as Campbell and Volteenaho (2004) have illustrated that *cash-flow beta* is the bad beta for the pro-cyclical asset as the payoff of pro-cyclical asset will turn the steam of investor's consumption more volatile, and it increases the consumption risk.

Moreover, *discount-factor beta* is used to measure the sensitivity between the revision of the conditional expectation of discount rate and the innovation of future business condition. Thus, high *discount-factor beta* follows the low unexpected rate of return because it is already shown in Equation (4) that uncertainty in the future economy is almost priced in the conditional discounted rate of return, and investor should not overpay the price of the asset.

Thus, we can see that high *discount-factor beta* is equivalent to less adjustment with the conditional expectation of discount rate, while investors can respond to innovation of future business condition rapidly without a large reversion in their conditional asset pricing models or setting too high a valuation to the asset. Hence, *discount-factor beta* is of good beta.

Similarly, if we replace the market innovation with aggregate consumption innovation, we can derive the components of risks for the consumption-based log-linear asset pricing model, and equation (6) becomes

$$\begin{aligned}\beta_{i,c} &\equiv \frac{Cov\left(\tilde{e}_i,\tilde{e}_c\right)}{Var\left(\tilde{e}_c\right)} = \frac{Cov\left(\tilde{e}_{d_{i,t+1}},\tilde{e}_c\right)}{Var\left(\tilde{e}_c\right)} - \frac{Cov\left(\tilde{e}_{r_{f,t+1}},\tilde{e}_c\right)}{Var\left(\tilde{e}_c\right)} - \frac{Cov\left(\tilde{e}_{r_{i,t+1}},\tilde{e}_c\right)}{Var\left(\tilde{e}_c\right)} \\ &\equiv \beta_{d_i,c} - \beta_{r_f,c} - \beta_{r_i,c}\end{aligned} \tag{7}$$

Equation (7) indicates the overall consumption beta as the sum of betas from each of the components with the innovations of aggregate consumption. To use this approach within a rolling regression framework, we can determine the importance of each component from the observed decrease of the overall beta in the long run.

Quite obviously, those distinct betas are highly associated with the variation in the consumption paths. And for pro-cyclical consumption based *cash-flow beta* mentioned above, it is the bad beta to turn the consumption steam more volatile, while the counter-cyclical consumption based *discount-factor beta* is of the good beta to smooth the consumption.

However, it is not clear as we observe if *cash-flow beta* and *discount-factor beta* should be tightly linked to the overall consumption risk of the equity. Furthermore, we would like to know why high equity premiums with value stocks and why value premium declines in the long horizons. We have, thus, conjectured that *cash-flow beta* is highly associated with the current macroeconomic risk, and it affects contemporary consumption growth more than the revision in expected future consumption growth in the long term.

Meanwhile, *discount-factor beta* could have played an important role with consumption smooth in the long run. It also exhibits that investors should revise their prior expectation of consumption growth regarding the long run mean of consumption growth as fast as they could.

To capture the marginal contributions of distinct sources of the risks, we have followed Epstein-Zin-Weil and used the separating risk aversion and inter-temporal substitution power utility to derive the beta identity of CCAPM.

Epstein-Zin-Weil objective function is defined recursively by

$$U_t = \left\{ \left(1-\delta\right) C_t^{\frac{1-\gamma}{\theta}} + \delta \left(E_t\left[U_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{\theta}} \right\} \tag{8}$$

where $\theta \equiv \left(1-\gamma\right)/\left(1-1/\psi\right)$, γ is the coefficient of relative risk aversion, ψ the elasticity of inter-temporal substitution. Risk aversion describes consumer's reluctance to substitute consumption across states of the world, and the elasticity of inter-temporal substitution shows the willingness of consumer to adjust their planned consumption growth in response to investment opportunities.

The inter-temporal budget constraint for the representative agent is

$$W_{t+1} = (1 + R_{w,t+1})(W_t - C_t) \tag{9}$$

where W_{t+1} is the representative agent's wealth at *t+1*, and $(1 + R_{w,t+1})$ the gross return on the wealth portfolio or market portfolio.

We can use dynamic programming to solve this problem and its Euler equation is

$$1 = E_t\left[\left\{\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\right\}^{\theta}\left\{\frac{1}{\left(1 + R_{w,t+1}\right)}\right\}^{1-\theta}\left(1 + R_{i,t+1}\right)\right] \tag{10}$$

If we assume that asset returns and consumption are homoskedastic and jointly lognormal, we use the property that

$$\log E_t\left[X\right] = E_t\left[\log\left(X\right)\right] + \frac{1}{2}Var_t\left[\log\left(X\right)\right] \tag{11}$$

The log-linear formula obtained for Euler equation is

$$\begin{aligned} 0 = {} & \theta\log\left(\delta\right) - \frac{\theta}{\psi}E_t\left[\Delta c_{t+1}\right] - \left(1-\theta\right)E_t\left[r_{w,t+1}\right] + E_t\left[r_{i,t+1}\right] \\ & + \frac{1}{2}\left\{\left(\frac{\theta}{\psi}\right)^2\sigma_c^2 + \left(1-\theta\right)^2\sigma_w^2 + \sigma_i^2 + 2\left[\frac{\theta\left(1-\theta\right)}{\psi}\sigma_{cw} - \frac{\theta}{\psi}\sigma_{ci} - \left(1-\theta\right)\sigma_{iw}\right]\right\} \end{aligned} \tag{12}$$

where $\Delta c_{t+1} \equiv \log\left(C_{t+1}\right) - \log\left(C_t\right)$ is the aggregate consumption growth, $r_{w,t+1} \equiv \log\left(1 + R_{w,t+1}\right)$ the gross return on the wealth portfolio, $r_{i,t+1} \equiv \log\left(1 + R_{i,t+1}\right)$ the gross return on the asset *i*, $\sigma_c^2 \equiv Var_t\left(\Delta c_{t+1}\right)$ the variance of the aggregate consumption growth, $\sigma_w^2 \equiv Var_t\left(r_{w,t+1}\right)$ the variance of the gross return on the wealth portfolio, $\sigma_i^2 \equiv Var_t\left(r_{i,t+1}\right)$ the variance of the gross return on asset *i*, $\sigma_{cw} \equiv Cov_t\left(\Delta c_{t+1}, r_{w,t+1}\right)$ the covariance of the aggregate consumption growth and the gross return on the wealth portfolio, $\sigma_{ci} \equiv Cov_t\left(\Delta c_{t+1}, r_{i,t+1}\right)$ the covariance of the aggregate consumption growth and the gross return on the and asset *i* , $\sigma_{iw} \equiv Cov_t\left(r_{i,t+1}, r_{w,t+1}\right)$ the covariance of the gross return on the wealth portfolio and the gross return on the asset *i*.

Since the conditional variance of risk-free asset and the co-variances between the risk-free asset return, consumption growth, and wealth portfolio return are equal to zero, the risk-free rate can be written as

$$r_{f,t+1} = \theta \log(\delta) - \frac{\theta}{\psi} E_t[\Delta c_{t+1}] - (1-\theta) E_t[r_{w,t+1}] + \frac{1}{2}\left\{\left(\frac{\theta}{\psi}\right)^2 \sigma_c^2 + (1-\theta)^2 \sigma_w^2 + 2\left[\frac{\theta(1-\theta)}{\psi}\sigma_{cw}\right]\right\} \quad (13)$$

If we substitute equation (13) into equation (12), we can obtain the premium on the risky assets, including the wealth portfolio

$$E_t[r_{i,t+1}] - r_{f,t+1} + \frac{\sigma_i^2}{2} = \theta \frac{\sigma_{ic}}{\psi} + (1-\theta)\sigma_{iw} \quad (14)$$

From equation (14), we find out that if $\theta \neq 1$, and the risk premium on risky asset is determined not only by its covariance with wealth portfolio but that of the consumption growth.

It is forthright for us to find that if the return on wealth is more volatile than consumption growth as implied by the use of a stock index return as proxy for the return on wealth, σ_{iw} may be much larger than σ_{ic}, and it may help to explain the *equity premium puzzle* or *value premium puzzle* in CCAPM.

To see how it works, we have supposed that the dividend on equity equals to the aggregate consumption multiplied by a measure of leverage λ

$$d_{et} = \lambda c_t \quad (15)$$

when $\lambda > 1$, dividends and stock returns are more volatile than the returns on the aggregate wealth portfolio.

In addition, equation (13) and (14) suggest a tight link between rational expectations of asset returns and of consumption, which is

$$E_t[r_{e,t+1}] = \mu_e + (1/\psi) E_t[\Delta c_{t+1}] \quad (16)$$

where μ_e is a constant term for long-term mean of consumption growth. The expected log return on equity or any other asset is but a constant plus expected consumption growth divided by the elasticity of inter-temporal substitution ψ.

To substituting equations (15) and (16) into equation (3), we can obtain

$$r_{e,t+1} - E_t[r_{e,t+1}] = \lambda(\Delta c_{t+1} - E_t[\Delta c_{t+1}]) + (\lambda - 1/\psi)(E_{t+1} - E_t)\sum_{j=1}^{\infty} \rho^j (\Delta c_{t+1+j}) \quad (17)$$

The unexpected log return on the wealth portfolio or market portfolio is λ times of contemporaneous unexpected consumption growth, plus $(\lambda - 1/\psi)$ times of the discounted sum of revision with the expected future consumption growth.

If we use equations (5), we can demonstrate that the asset beta between unexpected return of any asset and innovation in the consumption growth can be expressed as the sum of contemporaneous consumption beta and inter-temporal consumption beta

$$\begin{aligned}\beta_{e,c} &\equiv \frac{Cov(\tilde{e}_e,\tilde{e}_c)}{Var(\tilde{e}_c)} = \lambda\frac{Cov(\tilde{e}_c,\tilde{e}_c)}{Var(\tilde{e}_c)} + (\lambda - 1/\psi)\frac{Cov(\tilde{e}_c,\tilde{e}_{lc})}{Var(\tilde{e}_c)} \\ &\equiv \lambda + (\lambda - 1/\psi)\beta_{lc}\end{aligned} \tag{18}$$

where the first term λ is the parameter to measure the leverage of dividend's volatility relative to the aggregate consumption, which also suggests the macroeconomic risk of contemporaneous consumption. Without such generality, we can take it as *cash-flow beta.* Furthermore, the larger λ, the larger macroeconomic risk of contemporaneous consumption representative agent undertakes.

The second term $(\lambda - 1/\psi)\beta_{lc}$ can measure the long run risk. If $\lambda > 1/\psi$ and the inter-temporal covariance of the consumption $\beta_{lc} > 0$, the representative agent will suffer more long run risk, and he would need more risk premium on the equity. On the contrary, if $\lambda < 1/\psi$ and $\beta_{lc} > 0$ the representative agent would have a lower long run risk.

Empirically, several studies have discovered that the elasticity of inter-temporal substitution ψ is estimated to be close to zero in many countries (Campbell, 2003), and $\beta_{lc} > 0$ always holds because consumers increase their whole life consumption growths for consumption smoothing when their contemporaneous consumptions turn up unexpectedly.

For those reasons, we have thus anticipated that $(\lambda - 1/\psi)\beta_{lc}$ is impossible to be positive unless the leverage of dividend's volatility is extremely large, and implies that high equity premium should decline in the long horizons.

To learn all from one, we can see from the log-linear asset pricing framework, cash-flow beta will increase risk premium or market of beta of asset because of the pro-cyclical, while macroeconomic variation in asset's dividend will turn the steam of investor's consumption more volatile.

As a whole, the results from beta decomposition are consistent with the interpretation of conditional asset pricing analysis because they point out the direction of reduced cash flow risks.

4. Conclusion

The decrease in beta value and small stocks is interesting by itself, and it sheds light on the portfolio management widely used in empirical studies. However, the fact can have acquired even more relevance if it can be related to the debate on asset pricing anomalies.

In our model, we can find that current period values of cash flows depend on their exposure to macroeconomic risks.

Nonetheless, it is investors who cannot diversify their contemporary consumption risks, and what they need do is to revise their expectation of the cash flow growth and disperse consumption risks into different horizons. As such, it would induce another question: how is risk exposure priced in the long run?

According to Lettau and Watchter (2007), and Hansen et al. (2008), risk exposures of cash flows can be divided up by the gap between two points in time: the date of valuation and the date of the payoff. And it is considered to be of a potential resolution that can help us to understand the declining equity premium puzzle in the long run. Nonetheless, how such cash flows are priced when the gap in time becomes large?

The suggestions by Lettau and Watchter (2007), and Hansen et al. (2008) exhibit that statistical decompositions of cash flows are necessary to the analysis, and researcher should provide such decompositions with an economic model of valuation so as to consider the pricing of risk exposure in the long run comprehensively.

Besides, our model is found with an analogous idea, but with some difference. As Hansen et al. (2008) characterized the risk prices of cash flows; he had taken such a measure to show the long-run risk-return tradeoff for the valuation of cash flows exposed to fluctuations in macroeconomic growth. However, we have supplemented the financial model of valuation with decomposed multi-betas in order to measure the quantities of the risks.

Moreover, the long-run valuation of stochastic cash flows allows researcher to decompose long-run expected returns into the sum of a risk-free component and a long-term risk premium as found in our setting in equation (4). Thus, the long-term risk premium can be further decomposed into the product of a measure for long-run exposure to risk and the price of long-run risk.

In addition, our model is different from approaches that would examine relationship between one-period expected returns and preferences that feature concern about long-run risk (Bansal and Yaron, 2004; Campbell and Vuolteenaho, 2004; Bansal, 2007). On the contrary, we have focused on the inter-temporal consumption of risk, and, in particular, the implied risk quantities for cash flows far into the future.

In this paper, we try to find out the connection between the decrease in the beta of these portfolios and the emergence of premium in their expected return. Thus, we have hypothesized that conditional betas are the function of some economic state variables, and such assumption can help us distinguish the macroeconomic betas from the long run betas that drive decrease in equity premiums.

Besides, it is quite reasonable to believe that variation in macroeconomic conditions can affect the payoffs of risky assets and the expectation for future returns. Likewise, change of macroeconomic state variables also has influence on current consumption level as well as the forecast of intertemporal consumption growths.

Appendix A:

This appendix is derived from equation (2) - the expression for a log price-dividend ratio. First, we will reverse equation (26), and take the expectation on both sides to get

$$\begin{aligned} p_{i,t} &= k + E_t\left[\rho p_{i,t+1} + d_{i,t+1} - \rho d_{i,t+1} - r_{i,t+1}\right] \\ &= k + E_t\left[\rho\left(k + \rho p_{i,t+2} + d_{i,t+2} - \rho d_{i,t+2} - r_{i,t+2}\right) + d_{i,t+1} - \rho d_{i,t+1} - r_{i,t+1}\right] \\ &= \ldots\ldots \\ &= \frac{k}{1-\rho} + E_t\left[\lim_{j\to\infty}\rho^j p_{i,t+j} + d_{i,t+1} + \sum_{j=1}^{\infty}\rho^j \Delta d_{i,t+1+j} - \sum_{j=0}^{\infty}\rho^j r_{i,t+1+j}\right] \end{aligned}$$

Subtracting the current dividend on both sides, we can derive the equation (2).

Appendix B:

This appendix is derived from equation (17) -- the expression for the unexpected log return on any equity. To substituting equations (15) and (16) into equation (3), we can find that:

$$\begin{aligned} & r_{e,t+1} - E_t\left[r_{e,t+1}\right] \approx \left(E_{t+1} - E_t\right)\sum_{j=0}^{\infty}\rho^j \Delta d_{e,t+1+j} - \left(E_{t+1} - E_t\right)\sum_{j=1}^{\infty}\rho^j r_{e,t+1+j} \\ & = \left(E_{t+1} - E_t\right)\sum_{j=0}^{\infty}\rho^j \lambda \Delta c_{t+1+j} - \left(E_{t+1} - E_t\right)\sum_{j=1}^{\infty}\rho^j \frac{1}{\psi}\Delta c_{t+1+j} - 0 \\ & = \lambda\left(\Delta c_{t+1} - E_t\left[\Delta c_{t+1}\right]\right) + \left(\lambda - 1/\psi\right)\left(E_{t+1} - E_t\right)\sum_{j=1}^{\infty}\rho^j\left(\Delta c_{t+1+j}\right) \end{aligned}$$

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