# Determination of the IPO's Underwriting Price

# -Empirical Findings from A Proposed Fuzzy Games Model

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# Abstract

The primary goal of this study is to explore the initial public offering (IPO) pricing behavior in an underwriting system using fuzzy game theory. The profit goals of the underwriter and issuing company are established via linear programming. The fuzzy variable levels in a game are rationalized using interactive fuzzy planning. In this way, the most beneficial and appropriate offering price, for both underwriters and issuing companies, can be obtained. When the underwriter specifies the minimum level of satisfaction, how both parties can achieve maximum profit exchange satisfaction will be explored. Results of this study are: (1) In practice, the fuzzy game theory is more consistent with the human interference uncertainty relationship with independent and dependent variables using the profit return function as a potential function. Rewards for the underwriters can be judged according to the experience principle, which copes more accurately with actual situations. (2) Differences in satisfaction exist between underwriters and issuing companies for IPO pricing in the underwriting system. Profit or satisfaction exchanges can be conducted appropriately during the offering price negotiating process. (3) Change in profit and satisfaction sensitivity comparisons between the underwriters and issuing companies can serve as a reference for exchange and increase in satisfaction for underwriters and issuing companies.

# Key words: Pricing behavior; Underwriting system; Initial Offering Price (IPO); Fuzzy games theory

#### 1. Introduction

The frequent and significant excess return on initial public offerings (IPO) has been well documented. Most of the previous studies emphasized analysis of the reasons for discounted offering prices. Excellent accomplishments have been achieved with numerous basic theories with empirical evidence, such as the theory of asymmetrical information (Booth and James, 1996), the collateralization theory (Klein and Leffler, 1981), and the theory of special assets (Williamson, 1979),

Another major issue, however, involved in practical operations is the way to calculate a reasonable offering price. A final agreement has not yet been reached on this. One of the potential reasons is that the offering price evaluation model currently applied is not ideal as the existing relevant evaluation approaches are based on the financial ratio including the cash flow, price/earnings ratio (P/E ratio). Due to neglecting a critically important factor, there is a significant gap between the prices calculated using these methods and the finally actual offering price. In fact, offering prices are determined through negotiation between underwriters and issuing companies .

Some recent articles provide valuable outcomes for the pricing of IPO.Ellis, Michaely, and O'Hara(2000) examine aftermarket trading of underwriters and unaffiliated market makers in the three-month period after an IPO. They find that the lead underwriter is always the dominant market maker; he takes substantial inventory positions in the aftermarket trading, and co-managers play a negligible role in aftermarket trading. The lead underwriter engages in stabilization activity for less successful IPOs, and uses the overallotment option to reduce his inventory risk. Compensation to the underwriter arises primarily from fees, but aftermarket trading does generate positive profits, which are positively related to the degree of underpricing Madhavan(2000) finds the underwriter selects the spread with the objective of maxizing total revenues from commissions at the IPO stage plus future trading revenues.

Cho(2001) try to reconcile the average underpricing phenomenon with the expexted wealth maximizing behaviors of market participants in a best efforts system. Under the usual informational asymmetry, the optimal offer price for best efforts IPOs is derived as a function of the uncertainty about market's valuation, the expected return on proposed projects and the size of offerings relative to the firm's market value.

Ritter and Welch(2002) review the theory and evidence on IPO activity: why firms go public, why they reward first-day investors with considerable underpricing, and how IPOs perform in the long run. They find that many IPO phenomena are not stationary and the asymmetric information is not the primary driver of many IPO phenomena.

Aggarwal, Krigman, and Womack(2002) find managers usually do not sell any of their own shares in an initial public offering but instead wait until the end of the lockup period. We develop a model in which managers strategically underprice IPOs to maximize personal wealth from selling shares at lockup expiration. First- day underpricing generates information momentum by attracting attention to the stock and thereby shifting the demand curve for the stock outwards. This allows managers to sell shares at the lockup expiration at prices higher than they wouldotherwise obtain. We test the model on a sample of IPOs in the 1990s. We findthat higher ownership by managers is positively correlated with first-day underpricing, underpricing is positively correlated with research coverage, and research coverage is positively correlated with stock returns and insider selling at the lockup expiration. These results are consistent with the model. Biais, Bossaerts, and Rochet(2002) analyze the optimal IPO mechanism in a multidimentional adverse selection setting where institutional investors have private information about the market valuation of the shares, the intermediary has private information about the demand, and the institutional investors and intermediary collude. They find the uniform pricing is optimal ( all agents pay the same price) and characterizes the IPO price in term of conditional expectations.

Ljungqvist and Wilhelm(2003) find IPO underpricing or the first-day returns reached astronomical levels during 1999 and 2000. They show that the regime shift in initial returns and other elements of pricing behavior can be at least partially accounted for by marked changes in pre-IPO owner- ship structure and insider selling behavior over the period, which reduced key decision makers' incentives to control underpricing. After controlling for these changes, the difference in underpricing between 1999 and 2000 and the preceding three years is much reduced. They suggest that it was firm characteristics that were unique during the "dot-com bubble" and that pricing behavior followed from incentives created by these characteristics.

Bradley et al.(2004) investigate the pricing of 4,989equity IPOs with offer dates between 1981 and 2000. They find the issuing firm and its underwriter are more likely to negotiate from a set of rounded prices when the anticipated offer price is high and/or when there is a large degree of aftermarket price uncertainty. When a lower stock price and/or less uncertainty, negotiations should resolve a finer set of prices.

Derrien(2005) explores the impact of investor sentiment on IPO pricing. He use a model in which the aftermarket price of IPO shares depends on the information about the intrinsic value of the company and investor sentiment, I show that IPOs can be over- priced and still exhibit positive initial return. A sample of recent French offerings with a fraction of the shares reserved for individual investors supports the predictions of the model. Individual investors' demand is positively related to market conditions. Moreover, large individual investors' demand leads to high IPO prices, large initial returns, and poor long-run performance.

Corwin and Schultz(2005) examine syndicates for 1,638 IPOs from January 1997 through June 2002. We find strong evidence of information production by syndicate members. Offer prices are more likely to be revised in response to information when the syndicate has more underwriters and especially more co-managers. More co-managers also result in more analyst coverage and additional market makers following the IPO. Relationships be- tween underwriters are critical in determining the composition of syndicates, perhaps because they mitigate free-riding and moral hazard problems. While there appear to be benefits to larger syndicates, we discuss several factors that may limit syndicate size.

Once a new stock is listed, underwriters are most concerned about whether the offering price is consistent with the profit maximizing principle or not. An underestimated offering price may produce an ill will from the issuing company. Conversely, an overestimated offering price may result in an unmarketable situation, which creates a negative effect on the underwriter's revenue. Beatty and Welch (1996) pointed out that underwriters tend to reduce offering prices to the greatest extent, aiming to increase the capital return on subscription. However, issuing companies also anticipate maximum profit from an IPO. In addition to facing fixed direct cost expenses, issuing companies make great effort to increase the offering price. Consequently, both parties play a tug of war during the pricing process in attempt to determine the optimum offering price beneficial to both sides. The aforementioned typical evaluation approaches failed to take this coordination and negotiation mechanism into consideration.

Close observation of the behavior of underwriters and issuing companies during the course of pursuing the profit maximizing falls into the fuzzy game theory category to be discussed. Few previous studies relating the use of the fuzzy game theory to establish an IPO evaluation model were found. The purpose of this research is to apply and extend the fuzzy game theory to an IPO pricing process. In this study, we proposed a more complete asset evaluation approach than the existing methods to provide a better understanding of the pricing behavior of issuing companies and underwriters in the IPO market.

#### 2. Literature Review

#### 2.1. The IPO costs and returns for underwriters and issuing companies

There are two ways to underwrite stocks: best effort and firm commitment. The major income for the underwriters differs with the underwriting method. The main source of revenue for underwriters comes from service charges and commissions provided by the issuing company. However, in addition to the above sources, application processing brokerage commissions offer another source of income for underwriters.

Ritter (1987) divided underwriting costs into direct costs and discounted issuance price. Hansen (1992) considered the underwriting costs including the underwriter's profit, expenses listed on the prospectus, other expenditures not listed on the prospectus, negative return on stock prices resulting from capital increase, discount and option values. Chen and Ritter (2000) claimed that the difference in IPO offering price ranging around 7% in the 90s' and the offering prices depended on the financial and business situations of the issuing company as well as the issuance scale. Rewards for underwriters consist of discounted offering prices, underwriting fees, subscription rights, the right to match and consulting fees.

#### 2.2. The pricing methods in an underwriting system

Most of the previous studies relating to underwriting pricing systems involved practical evaluation models that include the free cash flow in a firm (FCFF), the (P/E) ratio and market customary practices. Rock (1986) applied pre-tax and after-tax risk-free interest rates to calculate the present free cash flow value in a firm. As estimating the after-tax risk-free cash flow market value, he divided the samples into: (1) fixed risk-free interest rate and company tax rate, (2) changeable risk-free interest rate and company tax rate, (2) changeable risk-free interest rate and company tax rate, and (3) floating risk-free interest rate and fixed company tax rate. He found that the market present value and adjusted after-tax risk-free interest rate cash flow values were the same. Kaplan & Richard (1995) adopted discounted cash flow (DCF) to examine very large margin trading loan transaction cases and found that the transaction price was close to the present predicted cash flow value. They considered that the cash flow approach was equally effective in estimating values as the P/E ratio could be used as a measurement tool for capitalized accounting.

Boatsman (1981) employed two different P/E ratio models to compare valuation precision. The first model randomly sampled companies in the same industry. The model located companies within the same trade with similar average earnings growth rates over ten years. The results indicated that the latter model was more precise. Alford (1992) applied the P/E ratio to evaluate the value of an enterprise and used three factors: the same industry, risk and earnings growth as the basis for

comparison. The results showed that the median predicting mean absolute error for the same industry factor was 24.5%, which was lower than 29.4% for the other two factors. This revealed that selecting similar companies from the same trade rendered a more precise valuation.

Customary practices on the market are pricing behavior in accordance with current regulations specified by the government. Some previous studies found that (1) P/E ratio, M/B (market to book) ratio and P/S (price to sale) ratio were good prediction factors and had a certain valuation effect, (2) estimated earnings were no better than historic earnings in prediction effect due to the uneven quality of predicting earnings for domestic listed companies, and (3) after adding growth and profitability into the P/S ratio approach, the interpretation capability and prediction accuracy were improved.

#### 2.3. Fuzzy games theory

From the perspective of game players, IPO pricing decision making behaviors can be divided into a cooperative game and a non-cooperative game. The main difference between these two games is if a binding agreement is reached between the persons involved while interacting. The game is a cooperative game because there won't be any resources left to improve the returns after the negotiating parties reach the Nash equilibrium. Conversely, both parties will compromise over the rewards to improve the game result and find Pareto optimal solution in a non-cooperative game. There are two-person, three-person and n-person game participants. The information that game participants hold can be divided into complete information and incomplete information. The former means that every participant has correct knowledge of the features, strategic space and payment functions of the other participants. The latter refers to an incomplete understanding of all information, which makes a specific answer impossible.

There are numerous implicit policymaker oriented preferences in practice such as decisions that require human judgment, discrimination and thought. This is especially true in game strategies and payment returns because non-quantified or vague characteristics exist like demand and resources. Therefore, the decision makers cannot precisely judge the possible objectives and limitations. Subjective policymaker judgments can be effectively added by applying the fuzzy game theory (Zadeh, 1965). The conditions then become more consistent with the conditions in reality. The combined fuzzy set and game theories is called fuzzy game theory (Lee et al., 1993).

When a game is played by two players that develop a leader-follower relationship, the dominant player will specify strategies. The follower will then establish his own profit objective after obtaining a full understanding of all of the information involving the profit objectives of the leader. This is called a Stackelberg game, which can be either cooperative or non-cooperative. Lai (1996) discussed Stackelberg games in which a follower fully cooperated with a dominant player. The follower would consider the dominant player's preferences and objectives and then use a satisfactory fuzzy goal specified by the dominant player to set up his own However, both sides might not be consistent with the level of satisfaction. dominant player's objective, achieving different levels of satisfaction. To circum this problem, Sakawa (1999) developed interactive fuzzy linear programming and separated fuzzy and non-fuzzy parameters to consider dominant and follower players' satisfaction levels. An IPO pricing model based on fuzzy game theory will be established in this study because the fuzzy game theory application range is

extensive and a study on underwriting system IPO pricing behavior applying this theory has not yet been conducted.

Chen and Larbani (2006) obtain weights of a multiple attribute decision making (MADM) problem with a fuzzy decision matrix by formulating it as a two-person zero-sum game with an uncertain payoff matrix.

Fang, Nuttle, and Wang (2004) develop a soft computing approach to maximize the seller's revenue in multiple-object auctions through the use of object sequencing.

# 3. Building Fuzzy Game Price Offering Model

#### 3.1. IPO pricing methods

The FCFF, P/E Ratio and customary market practices are the three methods used for stock offering pricing. FCFF considers the remaining disposable cash flow belonging to common stock shareholders after paying off the debt. As the discount rate is taken into consideration, it stands for a company risk variable-- an investment cost for internal shareholders and a required return on investment for the external stockholders. The discount rate (V) and the offering price (P) can be formulates as

$$V = \sum_{t=1}^{n} \frac{FFCF_{t}}{(1 + WACC_{t})^{t}}$$

$$P = \frac{V - Debt}{M}$$
(1)

*P*: offering price; *V*: cash flow; *WACC*: weighted average cost of capital; *M*: issued stocks

The corporate stock price equals the P/E ratio multiplied by the estimated earning per share (EPS). This is the most common calculation for general investors. It is based on the predicted growth rate, the dividend distribution rate and risk divided by the estimated stock value with the predicted net profit for P/E ratio. The predicted market value can then be calculated by multiplying it by the EPS for the following five years. The equation is

$$P_{0} = \frac{D_{0}(1+g)}{R_{s}-g} = \frac{D_{1}}{R_{s}-g}, R_{s} > g$$

$$\frac{P_{0}}{E_{1}} = \frac{D_{1}/E_{1}}{R_{s}-g} = \frac{d}{R_{s}-g}$$
(2)

 $P_0$ : stock price;  $D_0$ : dividend at term 0;  $D_1$ : dividend at period 1;

g: expected growth rate;  $E_1$ : earnings per share;  $R_s$ : risk level

The customary price offerings released by the authority can be expressed as

$$P = A \times 40\% + B \times 20\% + C \times 20\% + D \times 20\%$$
(3)

*P*: reference price offering; *A*: P/E ratio restored value; *B*: dividend restored value; *C*: net value per share in the previous year with 20% of the share offering rights; *D*: estimated dividend restored value; predicted interest on 1-year deposit for one dividend in financial institutions this year with 20% of the share offering rights

#### 3.2. Fuzzy game model for IPO pricing

The issuing company and underwriter are the two major parties in a public offering process. In addition to calculating the offering price in accordance with the customary market release practices, both parties will also resort to other underwriting methods like the P/E ratio to measure factors that cannot be quantified, such as goodwill, the company's prospects and market situations in the future. The pricing process between the underwriter and issuing company is a zero-sum game. As both parties have information on the corporate characteristics, strategies and returns for their counterpart, it is a cooperative game with complete information.

Five steps are taken for our analysis. First, we conduct return goal linear programming for both sides. Next, the policy making factors of both parties is fuzzified. Third, we design the service rate and lottery odds using fuzzy regression. This leads to the maximum and minimum fuzzy numbers. Fourth, we implement possible service rate and lottery odds distributions via the  $\alpha$ -cut approach. Fifth, we will establish the potential profit goals for the underwriter and issuing companies and seek consistent fuzzy goal satisfaction for both parties.

There are three basic hypotheses in this model: (1) Firm commitment is adopted. Underwriters may increase or decrease subscriptions to reduce risks. (2) Both parties are comparable while negotiating offering prices, which will be established in accordance with the profit maximizing principle for both sides. (3) the underwriter is the lead and the exclusive underwriter.

The variables relating to the price offering list model are defined as follows

- $\pi_{u}$ : profit function of the underwriter;
- $\pi_c$ : profit function of the issuing company;
- M: fair market price;
- $Q_s$ : total underwritten quantity;
- $Q_{\mu}$ : underwriter's subscription quantity;
- R: underwriter's accrued revenue from processing each application offer
- SP : processing fee expenditures of the issuing company;
- UC : revenue from the underwriter's subscription capital gains;
- UG: total underwritten amount;
- UL: revenue from drawing lots for subscription;
- UP : revenue from the underwriting processing fee;
- $\tilde{A}$ : drawing of lots for subscription represented by a fuzzy number;
- $\tilde{I}_u$ : underwriter handling fees presented using a fuzzy number; consulting and assisting fees, underwriting commissions are the major sources. Underwriting consulting and assisting fees depending on the company's price negotiating ability. This costs about \$500,000 to 1,000,000 for a case with a 10% public offering. The underwriting commissions account for 0.8% of the total funds raised in a public application offer.

Ritter (1987) considered that issuance costs could be divided into direct and indirect costs. In a zero-sum game, indirect cost refers to the cost of a discounted issuing company issuance, which may be deemed as revenue for other participants (underwriters and investors by drawing of lots). Therefore, a trade off occurs during the united profit maximizing calculation process in a cooperative game. However, this will not be necessary as calculating the profit functions for both parties. The major underwriter revenues come from processing fees, drawing of lots for subscription and capital gains on subscription. The underwriting revenues of issuing company come from capital gains on stock offerings, the market value of non-offered stocks. The primary costs consist of processing fees and underwriter commissions and the expenses listed on the prospectus such as accountant expenditures, counsel fees and handling fees.

#### A. Revenues from underwriting processing fees

It is specified in existing stock exchange regulations that the underwriting processing fee on a firm commitment basis should not exceed 5% of the aggregate When the total underwriting amount is large, the underwriter will sales. automatically reduce handling fees, which means that processing rates change with the total underwriting amount in a reverse way. The revenues from underwriting processing fees that the underwriter collects from the issuing company depend on the negotiated prices for the company and can be divided into consulting fees and underwriting commissions. Although processing rates become trivial as the total underwriting amount becomes larger for the underwriter, the revenues from application handling fees will increase. Therefore, the underwriter is willing to offer larger discounts on underwriting processing fees when the capital stock of the issuing company is huge. The underwriter will usually sacrifice commission to obtain more business. Underwriting commissions are subject to arbitrary judgment, which usually range from \$ 500,000 to \$1,000,000. Underwriters often reduce the rate charged to obtain potential future business opportunities.

The revenues from processing fees for underwriter are as follows:

Handling fee revenue (UP) is the handling fee rate ( $\tilde{I}_u$ ) multiplied by total underwriting amount (UG).

$$UP = I_{\mu} \times UG \tag{4}$$

The total underwriting amount (UG) is the price offered (P) multiplied by the total number of underwritten shares ( $Q_s$ ).

$$UG = P \times 1000 Q_s \tag{5}$$

As the handling fee rate is a fuzzy function, there are plenty of other subjective factors and market strategies in addition to the total underwriting amount that might affect handling fee rates. A probable distribution exists between the processing fee and the total underwriting amount, which means that they have an uncertain relationship on a bear market. The handling fee rate  $(\tilde{I}_a)$  is a non-random fuzzy number and can be formulated as

$$(\tilde{I})_{\alpha} = \left[ (\tilde{I})_{\alpha}^{L}, (\tilde{I})_{\alpha}^{R} \right], \quad (\tilde{a})_{\alpha} = \left[ (\tilde{a})_{\alpha}^{L}, (\tilde{a})_{\alpha}^{R} \right], \quad (\tilde{b})_{\alpha} = \left[ (\tilde{b})_{\alpha}^{L}, (\tilde{b})_{\alpha}^{R} \right]$$

The processing fee rate  $(\tilde{I})$  is negatively correlated to the total underwriting amount as follows

$$\tilde{I} = -\tilde{a} \times UG + \tilde{b} \tag{6}$$

After adding the human decision factor, it is represented by the fuzzy number  $\tilde{I}$ .  $\tilde{a}, \tilde{b}$  refer to the handling fee rate and total underwriting amount parameters. A negative correlation exists between the processing fee rate and the total underwriting amount, and then the processing fee revenue function (UP') is as follows:

$$UP' = \left[-\tilde{a} \times (P \times 1000 \ Q_s) + \tilde{b}\right] \times P \times 1000 \ Q_s \tag{7}$$

#### B. Processing fee expenditures for issuing companies

Issuing company must pay accountant certificate fees, underwriter counseling fees, underwriting commissions, printing expenses, announcement fees and other expenses. Accountant certificate fees depend on the company's bargaining power, which normally range from \$ 1,500,000 to \$2,000,000. Issuing 5,000 copies costs \$40 per copy for printing. The corporate business achievement announcement is determined by the size of the application offerings. The underwriting processing fee expenditures (SP) for the issuing company can be indicated as revenue from the underwriting processing fees (UP) added to a fixed constant (the certificate fees of accountants, the expenses of announcement, etc.).

$$SP = \left[-\tilde{a} \times (P \times 1000 \ Q_s) + b\right] \times P \times 1000 \ Q_s + c \tag{8}$$

where c is a constant (including accountants, lawyers, establishment fees)

#### C. Revenues from subscription by drawing lots

The relationship between the underwriter's profit and subscription by drawing lots can be deemed as market demand; i.e., when general investors anticipate differences between the offering and market prices, they would purchase offering prices that are discounted. This leads to reduced lottery odds. Therefore, a reverse relationship exists between the subscription by drawing lots and the discounted offering price. Wherein, the fuzzy function  $\tilde{A}$  stands for the lottery odds based on the experience principle required for pricing or professional judgments to measure the changes between the lottery odds and offering prices. The subscription processing fee revenues (UL) are the processing fee for one subscription actually received by underwriter multiplied by the total number of subscriptions.

According to Beatty and Welch (1996), the total number of subscriptions is a reciprocal of the offering price. Thus, the actual subscription number (LQ) is as follows:

$$LQ = \frac{(Q_s - Q_u)}{\tilde{A}}$$
<sup>(9)</sup>

The lottery odds interval of  $\tilde{A}$  is  $(\tilde{A})_{\alpha} = \left[ (\tilde{A})_{\alpha}^{L}, (\tilde{A})_{\alpha}^{R} \right]$ ,  $(\tilde{d})_{\alpha} = \left[ (\tilde{d})_{\alpha}^{L}, (\tilde{d})_{\alpha}^{R} \right]$ ,  $(\tilde{e})_{\alpha} = \left[ (\tilde{e})_{\alpha}^{L}, (\tilde{e})_{\alpha}^{R} \right]$ ,  $\tilde{d}, \tilde{e}$  are fuzzy parameters of the lottery odds and the discount respectively. The fuzzy lottery odds function can be shown as:

$$\widetilde{A} = -\widetilde{d} \times \frac{M - p}{M} + \widetilde{e}$$
(10)

 $\frac{M-p}{M}$ : discount rate of offering price;

 $\tilde{P}$  : fuzzy number for the offering price;

Revenue from the lot drawing subscription processing fees for underwriter(UL) is expressed as:

$$UL = 17.5 \times \frac{(Q_s - Q_u)}{-\tilde{d} \times \frac{M - P}{P} + \tilde{e}}$$
(11)

#### D. Number of shares subscribed by the underwriters themselves

Underwriters should subscribe all remaining unsold negotiable securities on a firm commitment basis upon the expiration of an underwriting. However, the minimum subscription rate cannot exceed 25% and subscribed stocks must be offered completely within a year. Beatty and Ritter (1986) claimed when the goodwill of an underwriter becomes an asset without any scrap value, one underwriting failure might lead to a total loss of the goodwill which the underwriter spared no pains to accumulate. To maintain this goodwill, the underwriter may increase the number of subscribed stocks on a firm commitment basis. The number of shares subscribed by underwriter is one of the variables studied in this study. When the capital gains from subscription are reduced in a bear market, the underwriter with abundant information will change the number of subscribed shares according to the stock market situation.

The capital gain (UC) from subscribed shares by underwriters equals the subscription number multiplied by the difference between the offered stock price and offering price.

$$UC = Q_u \times (M - P) \tag{12}$$

To calculate the fair market price, 30-, 90- and 180-day stock prices after listed are based and discounted in accordance with the proper discount rate at that time.

#### E. The profit function of underwriters and issuing companies

(A) The profit function  $(\pi_{\mu})$  of the underwriter is written as

$$\pi_{u} = UP + UL + UC$$

$$= [-\tilde{a} \times (P \times 100 Q_{s}) + \tilde{b}] \times P \times 100 Q_{s} + 17.5 \times \frac{(Q_{s} - Q_{u})}{-\tilde{d} \times \frac{M - P}{P} + \tilde{e}} + Q_{u} \times (M - P) + K$$
(13)

s.t.  $Q_s \leq Q_u$  $M \geq P \geq 10$ 

#### K : Fixed commission revenue.

The constraint  $M \ge P \ge 10$  means that when the market price is below the offering price, the underwriter will subscribe shares to prevent a capital loss.

(B) The profit function  $(\pi_c)$  of an issuing company is shown as

$$\pi_c = (M - 10) \times (Q_{all} - Q_s) \times 1000 + (P - 10) \times Q_s \times 100 - [-\tilde{a} \times (P \times 100Q_s) + \tilde{b}] \times P \times 100Q_s - c(14)$$

 $Q_{all}$ : the total number of issuance,

In a cooperative game, the united profits of an issuing company and underwriters are the sum of profits on both parties.

$$\begin{aligned} &Max \, \pi_c + \pi_u \\ = (M - 10) \times (Q_{all} - Q_s) \times 1000 + (P - 10) \times Q_s \times 1000 - c + 17.5 \times \frac{(Q_s - Q_u)}{-\tilde{d} \times \frac{M - P}{P} + \tilde{e}} + Q_u \times (M - P) + K \end{aligned} \tag{15}$$
  
s.t.  $Q_s \leq Q_u$   
 $M \geq P \geq 10$ 

The fuzzy number of united profits is  $[(\tilde{d})_{\alpha}^{L}, (\tilde{d})_{\alpha}^{R}], [(\tilde{e})_{\alpha}^{L}, (\tilde{e})_{\alpha}^{R}]$  and shown as  $\alpha$ -cut.

$$Max\pi_{c} + \pi_{u}$$

$$= (M-10) \times (Q_{all} - Q_s) \times 1000 + (P-10) \times Q_s \times 1000 - c + 17.5 \times \frac{(Q_s - Q_u)}{-(\tilde{d})^L_{\alpha} \times \frac{M-P}{P} + (\tilde{e})^U_{\alpha}} + Q_u \times (M-P) + K^{(16)}$$

F. The satisfaction and membership function for the underwriter and issuing company

The offering prices are negotiated by the underwriter and issuing company. Most of the underwriter's revenues comes from subscription by drawing lots and capital gains from the subscribed shares. This indicates that the underwriter will not give a higher offering price only for the sake of their own profits so that the capital gains from their subscribed shares after listing can be maximized. The weighted stock price index used to fall from 6100 to 3800 in a bear market. Over half of the IPO shares were lower than the offering price in 2002. According to Stock and Futures Committee (SFC) regulations, the offering prices for publicly issued valuable securities must be negotiated between the issuing company and underwriters in compliance with the announced pricing formula. The offering price, the number of underwritten shares and the number of subscribed shares will be determined during the negotiation process in accordance with the actual market As this model is based on the profit functions of both parties, situation. membership functions and the acceptable satisfaction on both parties can be calculated. In this way, changes in offering prices and number of shares subscribed by the underwriter can be explored when the level of satisfaction is limited or altered.

# (A) Definitions of the variables used in the model

 $P_n^{L}$ : the optimum offering price set by the underwriter;

 $P_n^{\mathcal{L}}$ : the optimum offering price set by the issuing company;

 $P_n^{L}$ : the least acceptable offering price set by the underwriter;

 $P_n^{L}$ : the least acceptable offering price set by the issuing company;

 $Q_n^{L}$ : the optimum number of shares subscribed by the underwriter;

- $Q_n^{L^*}$ : the optimum number of shares subscribed by the issuing company;
- $Q_n^{L-}$ : the least acceptable number of shares subscribed by the underwriter;
- $Q_n^{L^-}$ : the least acceptable number of shares subscribed by the issuing company;
- $f_u(p_n)$ : the function for change in profit in the underwriter's goal at the n-th time;
- $\mu_{f_{u}}(p_{n})$ : the membership function for the underwriter's goal at the n-th time;
- $f_c(p_n)$ : the profit change function for the issuing company under the n-th discount model;
- $\mu_{f_c}(p_n)$ : the membership function for the issuing company under the n-th discount model;
- $f_u^{L^*}$ : the least acceptable profit for the underwriter's profit function  $f_u(p_n)$ ;
- $f_{u}^{L}$ : the maximal profit for the underwriter's profit function  $f_{u}(p_{u})$ ;
- $f_c^{L}$ : the least acceptable profit for the issuing company's profit function  $f_c(p_x)$ ;
- $f_c^{t'}$ : the maximum profit for the issuing company's profit function  $f_c(p_n)$

(B) The profit change function for the underwriter can be written as

$$[-\tilde{a} \times (P \times 1000Q_{1s}) + \tilde{b}] \times P \times 1000Q_{1s} + 17.5 \times \frac{(Q_{1s} - Q_{1u})}{-\tilde{d} \times \frac{M - P_1}{P_1} + \tilde{e}} + Q_{1u} \times (M - P_1) + K$$
(17)

 $Q_{ls}$ : the number of changes in the underwriting shares for the first time;

 $P_1$ : the first offering price change after the  $\alpha$ -cut;

(C) The profit change function for the issuing company is expressed as

$$(M-10) \times (Q_{all}-Q_{1s}) \times 1000 + (P_{1}-10) \times + Q_{1s} \times 1000 - [-a \times (P_{1}\times 1000Q_{1s}) + b] \times P_{1} \times 1000Q_{1s} - c_{1s} \times 1000Q_{1s} + c_{1s} \times 1000Q_{1s} - c_{1s} \times 1000Q_{1s} + c_{1s} \times 1000Q_{1s} - c_{1s} \times 1000Q_{1s} + c_{1s} \times 1000Q_{1s} - c_{1s} \times 100Q_{1s} + c_{1s} \times 10Q_{1s} + c_{1s} \times 10Q_{1s} + c_{1s} \times 10Q_{1s} + c_{1s} \times 10Q$$

(18)

When both parties negotiate offering prices, change in the offering prices results in a fuzzy lottery odds and a handling fee distribution. When the price negotiation over the offering price is reached to reduce the price, the discount rate becomes smaller and lottery odds increase. This leads to a decrease in subscription revenue for the underwriter. Moreover, an increase in the total underwriting amount causes the handling fees to fall. This results in a decrease in handling fee revenue for the underwriter. A reduction in the underwriter's revenue implies increasing profit for the issuing company. Therefore,  $\alpha$  determines increase/decrease in total profits for both parties.

(D) The maximum and minimum profit functions for the underwriter and issuing company

The least acceptable and most satisfactory profit goals for an underwriter exist among underwriting profit functions. The least acceptable profit goal for an underwriter is the profit function  $f_u^{L-}$  and  $f_{n-1}^{L-}$  is defined as the best profit after the n-1 th discount and change in underwriting amount:

$$f_{n-1}^{L-} = \left[-\tilde{a} \times (P_{n-1}^{L-} \times 1000Q_{n-1s}^{L-}) + \tilde{b}\right] \times P_{n-1}^{L-} \times 1000Q_{n-1s}^{L-} + 17.5 \times \frac{(Q_{n-1s}^{L-} - Q_{n-1u}^{L-})}{-\tilde{a} \times \frac{M - P_{n-1}^{L-}}{P_{n-1}^{L-}} + \tilde{e}} + Q_{n-1}^{L-} \times (M - P_{n-1}^{L-}) + K$$
(19)

If the maximum profit function  $(f_u^{L^*})$  for an underwriter is written as

$$f_n^{L^*} = \left[-\tilde{a} \times (P_n^{L^*} \times 1000Q_{ns}^{L^*}) + \tilde{b}\right] \times P_n^{L^*} \times 1000Q_n^{L^*} + 17.5 \times \frac{(Q_{ns}^{L^*} - Q_{nu}^{L^*})}{-\tilde{d} \times \frac{M - P_n^{L^*}}{P_n^{L^*}} + \tilde{e}} + Q_n^{L^*} \times (M - P_n^{L^*}) + K$$
(20)

then the least acceptable profit function  $(f_s^L)$  for the issuing company is formulated as

$$f_{n-1}^{L-} = (M - 10) \times (Q_{all} - Q_{n-1s}^{L-}) \times 1000 + (P_{n-1}^{L-} - 10) \times Q_{n-1s}^{L-} \times 1000 - [-\tilde{a} \times (P_{n-1}^{L-} \times 1000 Q_{n-1s}^{L-}) + \tilde{b}] \times P_{n-1}^{L-} \times 1000 Q_{n-1s}^{L-} - c$$
(21)

The maximum profit function  $(f_s^{L^s})$  for the issuing company is

$$f_n^{L^*} = (M - 10) \times (Q_{all} - Q_{ns}^{L^*}) \times 1000 + (P_n^{L^*} - 10) \times +Q_{ns}^{L^*} \times 1000 - [-\tilde{a} \times (P_n^{L^*} \times 1000 Q_{ns}^{L^*}) + \tilde{b}] \times P_n^{L^*} \times 1000 Q_{ns}^{L^*} - c$$
(22)

### (*E*) The membership of profit function for the underwriter and issuing company The membership of profit function for an underwriter is

$$\mu_{u}(p_{n}) = \begin{cases} 0 & \text{for } f_{u}(p_{n}) < f_{u}^{L^{-}} \\ \frac{f_{u}(p_{n}) - f_{u}^{L^{-}}}{f_{u}^{L^{*}} - f_{u}^{L^{-}}} & \text{for } f_{u}^{L^{-}} < f_{u}(p_{n}) < f_{u}^{L^{*}}. \\ 1 & \text{for } f_{u}(p_{n}) > f_{u}^{L^{*}} \end{cases}$$
(23)

The membership of profit function for the issuing company is formulated as

$$\mu_{s}(p_{n}) = \begin{cases} 0 & \text{for } f_{s}(p_{n}) < f_{s}^{L^{-}} \\ \frac{f_{s}(p_{n}) - f_{s}^{L^{-}}}{f_{s}^{L^{*}} - f_{s}^{L^{-}}} & \text{for } f_{s}^{L^{-}} < f_{s}(p_{n}) < f_{s}^{L^{*}} \\ 1 & \text{for } f_{s}(p_{n}) > f_{s}^{L^{*}} \end{cases}$$

$$(24)$$

(F) The optimum solution for the maximum limitation and the minimum goal (Bellman and Zadeh, 1965):

$$FG_{\max \min} = \text{Max } \mu_{FG}(P) = Max \left[\mu_{C}(p) \land \mu_{G}(p)\right]$$
(25)  
$$FG_{\max \min} : \text{the solution to the optimum fuzzy goal}$$

 $\mu_{g}(p)$ : the membership function for the profit goal for the underwriter

 $\mu_{c}(p)$ : the membership for the limited profit function for the underwriter.

Where suppose  $\alpha$  is the fuzzy number for any solution,  $\alpha \in [0,1]$  and  $\beta$  is the satisfaction value established by the underwriter's profit goal,  $\beta \in [0,1]$ .  $(\alpha,\beta)$  are the potential goal, limit and fuzzy function combinations. The best combination is to locate  $_{MaxMin(\alpha,\beta)}$ . Now, if  $\lambda = \min(\alpha,\beta)$ , then it can be written as:

$$Max \qquad \lambda$$

$$s.t. \qquad \lambda \leq \alpha , \ \lambda \leq \beta ,$$

$$W_{u} \left( \frac{f_{u} \left( p_{n} \right) - f_{u}^{L^{-}}}{f_{u}^{L^{+}} - f_{u}^{L^{-}}} \right) \geq \beta ,$$

$$W_{c} \left( \frac{f_{c} \left( p_{n} \right) - f_{c}^{S^{-}}}{f_{c}^{S^{+}} - f_{c}^{S^{-}}} \right) \geq \beta ,$$

$$\beta \in (0,1), W_{u} + W_{c} = 1$$

$$(26)$$

and

$$f_{u}^{L} = \left[-(a)_{\alpha}^{L} \times (P \times 1000Q_{s}) + (b)_{\alpha}^{R}\right] \times P \times 1000Q_{s} + 17.5 \times \frac{(Q_{s} - Q_{u})}{-(d)_{\alpha}^{L} \times \frac{M - P}{P} + (e)_{\alpha}^{R}} + Q_{u} \times (M - P) + K \quad (27)$$

$$f_s^L = (M - 10) \times (Q_{all} - Q_s) \times 1000 + (P - 10) \times +Q_s \times 1000 \qquad -[-(a)_{\alpha}^L \times (P \times 1000Q_s) + (b)_{\varepsilon}^R] \times P \times 1000Q_s - c$$
(28)

#### 4. An Empirical Model

#### 4.1. Data Source and Selections of Variables

The purpose of this research is to explore if the negotiation between IPO companies and underwriters over the offering prices is based on the profit maximizing principle for shareholders. To eliminate the uncertainty of offering price risk resulting from market situations, a study on the newly listed and over-the counter (OTC) shares issued from January 1, 2002 to September 28, 2002 in Taiwan bear market is conducted. Lottery odds and handling fee samples for the underwriters are from 14 IPO listing and 73 OTC issuing companies. This makes the total number 84. After ignoring 8 companies without issuing application offers by lots, the valid sample number is 76. The standard deviations dispersed tremendously since the market value is below the offering price. This leads to extreme fluctuations in the dependent and independent lottery odds, discount, handling fee and total underwriting amount variables. Therefore, adopting fuzzy regression instead of statistical regression rendered better results (Kim 1999). The reference prices calculated from the offering prices and formulas were selected from the IPO prospectus of issuing companies. The processing fee revenues and lottery odds calculation for all securities underwriters were collected. The actual market price is calculated based on the Taiwan Economic Journal (TEJ) database. The fair market price is the average price from the 31st to 60th days after listing with the proper discount. The discount interest rate is based on the interest rate for 3-month fixed deposits from the same period. The objects in this study are Company A, Company B and Company C (see Table 1 for respective profile).

			-
Company	Company A	Company B	Company C
Company	(biotechnology)	(electronic)	(electronic)
Capital	\$948,332,000	\$2,168,666,210	\$688,565,830
Total Shares	94,833,200 shares	216,866,621 shares	68,856,583 shares
Underwriter	Grand Cathay	National Investment	Taiwan International
Underwitter	Securities Corp.	Trust Co., Ltd.	Securities Corporation
Term of Issuance	Full public issuance	Full public issuance	Full public issuance
Rate of Public Offering	15.35% of listed shares 14,555,000 shares in total	Dispersed share rights; 20 million shares for public underwriting with a rate of 9.22%	10% of listed shares 6,885,658 shares in total
Underwriting Subscription	Underwriter subscription and public subscription on a firm commitment basis	Underwriter subscription and public subscription on a firm commitment basis	Underwriter subscription and public subscription on a firm commitment basis
Major Commodity	Medical appliances for rehabilitation (electric scooters, electric wheelchairs)	PC-based peripheral ICs, high-level consumer ICs, other application ICs	Optical fiber, microwave communication, data communication equipment
Offering Price	\$60 per share	\$278 per share	\$25 per share
Reference Price	\$41.24 per share	\$328 per share	\$17.63 per share
EPS in 2002	\$3.39	\$15.34	\$2.19
Estimated EPS in 2003	\$4.25	\$10.10	\$1.09
Main Comparable Industry	Mechanical & electrical companies, chemical industry, all listed stocks	VIA Technologies Inc., Sunplus Technology Co., Ltd., Realtek Semiconductor Corp.	Listed companies

Table 1 Profiles of Issuing Companies

# 4.2. Building a Empirical Model

The IPO bull run is projected at one month and the fair market value is based on the average daily closing price for the sample companies after listing for a month. This is compared with the offering price for the average closing price. After discounting the market return rate for the listing year (3-month fixed deposit interest rate of the Bank of Taiwan), the market value of Company A is \$102.7, that for Company B \$260 and that for Company C \$27.9.

Subscription by drawing lots is a potential function and stands for the relationship between the discount and lottery odds. Parameters  $\tilde{d}, \tilde{e}$  are fuzzy triangular numbers, -0.0522 and 0.2296 represent the  $\tilde{d}, \tilde{e}$  parameter, respectively. The values of 0.1466 and 0.073 are the parameter widths  $\tilde{d}, \tilde{e}$  and  $\alpha$  are a grace value subject to the rate of the function. In a regression where the inputs are specific outputs of the fuzzy numbers, Tanaka (1982) considered the tolerable value for the fuzzy goal should reach  $\sum_{i=1}^{M} 2e_i$  at least ( $e_i$  as dispersion value of item i); i.e., the output range of two standard deviations. Because 2002 was a bear market, calculating the standard deviation of the lottery odds is based on the lottery odds for new listings in the same year, reflecting the opinions of general investors in the stock market and a price comparison. The lottery odds statistics for the sample companies are shown in Table 2. The average lottery odds for that year are 22.86 with a standard deviation of 36.38. This indicates that the investors are not willing to subscribe in a bear market with relatively higher underwriting offer lottery odds.

	Od	lds of Lotter	ry	Discount		
Year	Maximum	Minimum	Average	Maximum	Minimum	Average
2002	100	0.46	22.86	198.7	-80.5	9.63
	]	Handling Fe	ee	Total Unde	erwriting Amo	ount (\$1,000)
2002	14.29	0.07	1.78	5,560,000	4,008	231,949

 Table 2
 Statistics of Odds of Lottery and Handling Fees

Note: Unit: %

The handling fee rate shows a reverse relationship between handling fee and the total underwritten amount. According to SFC regulations, the underwriting handling fee for capital less than a billion dollars cannot exceed 5%. Underwriters usually give discounted handling fees to acquire more business in a bear market. In 2002, the average underwriting handling fee was 1.78%, which was lower than normal. The standard deviation was 1.75%, which indicates that the differences in handling fee rates are large.

For the number of underwritten and subscription shares by underwriters, the SFC specifies a 20% underwriting rate for listing capital less than 1 billion, 15% for listing capital from one billion to two billion, 10% for listing capital from 2 billion to 5 billion and over 10% for OTC listings. Table 3 indicates that most of the issuing companies adopt a 10% underwriting rate for public issuance.

	Underwriting Rate (%)			Subscription	n Rate by Und	erwriters (%)
Year	Max.	Min. Average		Max.	Min.	Average
2002	10	7.68	9.85	100	2.5	27

Table 3 Statistics of Underwriting Shares and Subscription Shares by Underwriters

Note: Unit: share

#### 4.3. Empirical Results an Analysis

#### A. The price on pricing models

For pricing valuation, the discounted cash flow pricing results approach (Table 4),  $\alpha - cut$  of the customary practice in the market (Table 5) and the  $\alpha - cut$  of P/E approach (Table 6) revealed that underwriter handling fee revenue decreased as the grace value ( $\alpha$ ) increased. As an increase in  $\alpha$  leads to an increase in handling fee and the handling fee rate is inversely proportional to the total underwritten amount, the total underwritten amount is reduced and the processing fee revenue decreases as well. An increase in  $\alpha$  could result in increase in lottery odds, which not only renders a reduction in the total number of application offerings and processing fee revenue, and also a decrease in total underwriting profit. Conversely, the total profit for the issuing company increases along with the fuzzy value.

						U	Jnit: \$1,000
	2001	2002	2003	2004	2005	2006	2007
Working Capital Cash Flow	414,862						
Working Capital Change	6,578						
Capital Expenditure	257,007						
Disposable Cash Flow	151,277	161,388	172,174	183,681	195,958	209,054	223,026
Equity Capital Cost	6.4687	6.4687	6.4687	6.4687	6.4687	6.4687	6.4687
Discount Rate	1.0647	1.0647	1.0647	1.0647	1.0647	1.0647	1.0647
Value during Valuation Period	151,277	151,582	151,888	152,194	152,502	152,809	
Value of Sustainable Operation							2,520,169
Value of Common Stock	3,432,421						
Shares Issued (1,000 shares)	72,774						
Value per Share	47.165						

Table 4 Discounted Cashflow Pricing Approach – Company A

Note 1: No preferred stock and debenture stock were issued by 2002. The average weighted capital cost was the equity capital cost, which was from the prospectus.

Note 2: Working capital cash flow of Company A was based on average geometric historic data with a growth rate of 6.6835%.

Note 3: Sustainable operation value is based on Gordon model  $\frac{FCF_t(1+g)}{(K_e-g)}$  with g as

the growth rate and K as the discount rate.

α level	Offering Price (\$)	Subscription Number By Underwriter	Underwriting Handling Fee Revenue (\$)	Underwriter Subscription Handling Fee Revenue (\$)	Total Profit of Underwriter (not including subscription by underwriter) (\$)	Total Profit of Issuing Company (\$)
1	41.24	13,099	10,208,388	1,342,187	12,050,575	11,323,300,072
0.9	41.24	13,099	10,005,988	1,224,812	11,730,800	11,323,502,472
0.8	41.24	13,099	9,803,589	1,126,315	11,429,904	11,323,704,871
0.7	41.24	13,099	9,601,190	1,042,480	11,143,670	11,323,907,270
0.6	41.24	13,099	9,398,791	970,261	10,869,052	11,324,109,669
0.5	41.24	13,099	9,196,392	907,400	10,603,792	11,324,312,068
0.4	41.24	13,099	8,993,993	852,189	10,346,181	11,324,514,467
0.3	41.24	13,099	8,791,593	803,311	10,094,904	11,324,716,867
0.2	41.24	13,099	8,589,194	759,735	9,848,930	11,324,919,266
0.1	41.24	13,099	8,386,795	720,644	9,607,439	11,325,121,665
0	41.24	13,099	8,184,396	685,379	9,369,775	11,325,324,064

Table-5  $\alpha$ -cut Value of Customary Practice on Market – Company A

α Level	Offering Price (\$)	Subscription Number by Underwriter	Underwriting Handling Fee Revenue (\$)	Underwriter Subscription Handling Fee Revenue (\$)	Total Profit of Underwriter (not including subscription by underwriter) (\$)	Total Profit of Issuing Company (\$)
1	63.07	13,099	15,610,931	1,035,327	17,146,258	11,603,848,699
0.9	63.07	13,099	15,301,537	1,025,367	16,826,904	11,604,158,093
0.8	63.07	13,099	14,992,143	1,015,597	16,507,740	11,604,467,487
0.7	63.07	13,099	14,682,748	1,006,011	16,188,760	11,604,776,882
0.6	63.07	13,099	14,373,354	996,605	15,869,959	11,605,086,276
0.5	63.07	13,099	14,063,960	987,373	15,551,333	11,605,395,670
0.4	63.07	13,099	13,754,566	978,310	15,232,877	11,605,705,064
0.3	63.07	13,099	13,445,172	969,413	14,914,585	11,606,014,458
0.2	63.07	13,099	13,135,778	960,675	14,596,453	11,606,323,852
0.1	63.07	13,099	12,826,384	952,094	14,278,478	11,606,633,246
0	63.07	13,099	12,516,990	943,665	13,960,654	11,606,942,640

Table 6  $\alpha$ -cut Value of P/E Ratio Approach – Company A

B. Profits of underwriters and Issuing companies

Based on (13) and (14), the profits of underwriters and issuing companies are reported in Table 7

	Joinpung			
	Day 60			
	Maximum Profit Of Underwriter	Minimum Profit Of Underwriter	Maximum Profit Of Issuing Company	Minimum Profit of Issuing Company
Company A	371,201,641	30,733,175	14,771,964,946	13,318,642,269
Company B	2,632,389,951	52,060,108	56,980,260,892	54,399,931,050
Company C	18,370,846	3,043,653	648,113,490	599,936,556
	Day 180			
Company A	516,364,701	21,130,259	9,510,879,527	8,574,708,429
Company B	6,902,314,963	127,366,838	149,507,163,162	142,732,215,037
Company C	NA	NA	NA	NA

Table 7Statistics of Maximum and Minimum Profits of Underwriters and Issuing<br/>Company

	Day 360			
Company A	379,893,961	30,933,403	15,554,215,197	14,022,451,539
Company B	NA	NA	NA	NA
Company C	13,021,501	2,696,784	434,947,395	402,491,356

Note: Unit: \$

# C. Membership of fuzzy goals for the underwriter and membership of fuzzy goal for Issuing Company

The fuzzy goal of an underwriter is  $V_{u}(p_{1}) = [(-0.0049 + 0.0006(1 - \alpha)) \times (PQ_{s}/10000000) + (0.01868 - 0.00376(1 - \alpha))] \times P \times 1000Q_{s} + 17.5 \times \frac{(Q_{s} - Q_{u})}{(-0.0522 + 0.1466(1 - \alpha)) \times \frac{M - P}{P} + (0.2296 + 0.073(1 - \alpha))} + Q_{u} \times (M - P) + K$ (29)

The membership of a fuzzy goal for the underwriter is as follows

$$\mu_{u}(p_{1}) = \begin{cases} 0 & \text{for } V_{u}(p_{1}) < 30733175 \\ \frac{V_{u}(p_{1}) - 30733175}{371201641 + 30733175} & \text{for } 30733175 \le V_{u}(p_{1}) \le 371201641 \end{cases}$$
(30)

The membership of a fuzzy goal for issuing company can be expressed as

$$\mu_{c}(p_{1}) = \begin{cases} 0 & \text{for } V_{c}(p_{1}) < 1318642269 \\ \frac{V_{c}(p_{1}) - 1318642269}{1477196496 - 1318642269} & \text{for } 1318642269 \\ V_{c}(p_{1}) \le 1477196496 \\ \text{for } V_{c}(p_{1}) \ge 1477196496 \end{cases}$$
(31)

The statistical relationship between the offering price and profit satisfaction is based on different approaches listed in Tables 8 through 10. Optimum prices of \$42.6, \$45.8 and \$45.3 appeares on Days 60, 180 and 360 for Company A. Using Company A as an example, the prices calculated using three different models were \$41.24 for the customary market practice, \$63.07 for the P/E ratio approach and \$47.17 for the FCFF. This indicates that the customary market practice price is the lowest and that for the P/E ratio approach was the highest. In fact, the offering price of \$60 per share provided by the issuing company is closer to the P/E ratio price However, the optimum price of \$42.6 is closer to \$41.24 for the approach. customary market practice. Even when underwriters can precisely judge the economic or industry trends and calculate the optimum price, they may be unable to convince issuing companies to take the optimum price or choose a valuation model consistent with the maximum profit. For this reason, the market price is lower than the offering price for Companies B and C. There is no IPO bull run for Company C and a large difference exists between the finalized offering price of \$25 and three optimal prices of \$14.75, \$17.63 and \$13.2 calculated in three different periods.

This result indicates that the issuing company overestimates the P/E ratio for its industry while pricing. The \$17.63, \$20.26 and \$33.92 prices are calculated using three different approaches; the customary market practice price is closer to the optimum price than the other values. The final offering price is more consistent with the P/E ratio price approach. This shows that the issuing company is more

competent in bargaining power. The  $\beta$  values are 0.2885, 0.4911 and 0.2962 at different offering points, respectively. This reveals a lower membership compared with Companies B and C. The discount and the number of underwriter subscriptions are two major factors influencing profit satisfaction. For issuing companies, fewer discounts means higher offering price, which renders greater profit to the companies.

For underwriters, the subscription ratio may be increased or decreased in accordance with the company's' future value. To increase profit satisfaction for both sides in the example of Company A, the underwriters may increase the number of subscriptions from 1456 to 3256 with an optimum price of \$66.6 and a changed  $\beta$  value of 0.500. The optimum price would be close to the offering price at this moment. The  $\beta$  value for the underwriter on Day 60 is 0 in the Company B example and the prices calculated using three approaches were \$328, \$364.3 and \$391.5, which are all higher than the market price of \$278. The underwriter obtained handling fee revenue only because of the hedge measures taken. The market price increases to \$700 on Day 180 with a  $\beta$  value of 0.5, an optimum price of \$355, an actual offering price of \$278,  $\beta$  value of 0.6116 for the underwriter and a  $\beta$  value of 0.3884 for the public company. The offering price can increase to promote profit satisfaction for the issuing company and the P/E ratio approach can achieve the maximum profit. Company B is an example of full subscription.

As the actual offering price is far lower than the prices calculated using three different valuation models, issuing companies are less satisfied with the profit than the underwriters. Under full subscription, the major revenue for underwriters comes from the capital gains after the IPO bull run. Although the stock price of Company B after two-months listing is lower than the actual offering price, and dropped even more after six months; the stock price of Company B eventually rises to \$ 700. Either is the underwriter trading insider or drives up the stock price to gain maximum profit. Determining what actually occurred requires longer term observation. The optimum prices of Company C on Day 60 and 369 are \$14.75 and \$13.2, respectively. The optimum pricing approach adopted is the customary market practice. Because the price is below the offering price, the underwriter could not adjust the subscription ratio, and only take hedge to prevent a subscription loss. The stock prices and optimal prices for the above companies are summarized in Table 11.

			,		
Offering Price (\$)	No. of Subscription by an Underwriter	$\beta$ Value of an Underwriter	Profit of an Underwriter (\$)	Profit of an Issuing Company (\$)	$\beta$ Value of an Issuing Company
Day 60					
42.6 (Optimum Price)	1,456	0.2885	128,919,940	13,737,919,244	0.2885
41.24 (Customary Practice on Market)	1,456	0.2933	130,592,008	13,720,427,931	0.2765
60 (Actual Offering Price)	1,456	0.2258	107,603,199	13,961,705,176	0.4425
63.07 (P/E Ratio)	1,456	0.2149	103,899,412	14,000,674,799	0.4693
47.17 (FCFF)	1,456	0.2712	123,309,888	13,796,695,200	0.3289

Table 8Profit Satisfaction  $\beta$ Value of Company A

Day 180					
45.8 (Optimum Price)	1,456	0.4911	66,282,893	9,035,141,434	0.4918
41.24 (Customary Practice on Market)	1,456	0.5518	71,862,857	8,976,494,091	0.4292
60 (Actual Offering Price)	1,456	0.3024	48,934,401	9,217,771,336	0.6869
63.07 (P/E Ratio)	1,456	0.2617	45,187,035	9,257,255,410	0.7291
47.17 (FCFF)	1,456	0.4729	64,607,578	9,052,761,360	0.5106
Day 360					
45.3 (Optimum Price)	1,456	0.2962	134,316,621	14,476,453,914	0.2964
41.24 (Customary Practice on Market)	1,456	0.3106	139,308,251	14,424,237,201	0.2623
60 (Actual Offering Price)	1,456	0.2447	116,308,392	14,665,514,446	0.4120
63.07 (P/E Ratio)	1,456	0.2339	112,554,797	14,704,998,520	0.4456
47.17 (FCFF)	1,456	0.2897	132,021,095	14,500,504,470	0.3120

# Table 9Profit Satisfaction $\beta$ Value of Company B

Offering Price (\$)	No. of Subscription by an Underwriter	$\beta$ Value of an Underwriter	Profit of an Underwriter (\$)	Profit of an Issuing Company (\$)	$\beta$ Value of An Issuing Company
Day 60		_			
141.4 (Optimum Price)	10,000	0.4999	1,342,156,626	55,690,164,374	0.5000
328 (Customary Practice on Market)	10,000	0	50,021,641	57,522,329,215	1
278 (Actual Offering Price)	10,000	0	50,021,641	57,031,392,889	1
364.3 (P/E Ratio)	10,000	0	50,021,641	57,878,750,159	1
391.5 (FCFF)	10,000	0	50,021,641	58,145,821,043	1
Day 180				•	
355 (Optimum Price)	10,000	0.5000	3,514,885,392	146,119,644,608	0.5000
328 (Customary Practice on Market)	10,000	0.5391	3,779,991,785	145,854,538,215	0.4609
278 (Actual Offering Price)	10,000	0.6116	4,270,928,111	145,363,601,889	0.3884
364.3 (P/E Ratio)	10,000	0.4866	3,423,570,841	146,210,959,159	0.5135
391.5 (FCFF)	10,000	0.4471	3,156,499,957	146,478,030,043	0.5529
Day 360					
NA (Optimum Price)	10,000	NA	NA	NA	NA
328 (Customary Practice on Market)	10,000	NA	NA	NA	NA
278 (Actual Offering Price)	10,000	NA	NA	NA	NA
364.3 (P/E Ratio)	10,000	NA	NA	NA	NA
391.5 (FCFF)	10,000	NA	NA	NA	NA

Offering Price (\$)	No. of Subscription by an Underwriter	$\beta  Value \\ of an \\ Underwriter$	Profit of an Underwriter (\$)	Profit of an Issuing Company (\$)	$\beta$ Value of an Issuing Company
Day 60					
14.75 (Optimum Price)	1,721	0.4995	10,699,724	624,025,041	0.5000
17.63 (Customary Practice on Market)	1,721	0.1966	6,056,781	638,630,267	0.8032
25 (Actual Offering Price)	1,721	0	3,043,615	676,005,456	1
20.26 (P/E Ratio)	1,721	0	3,043,615	657,961,680	1
33.92 (FCFF)	1,721	0	3,043,615	721,241,115	1
Day 180	I		I	l .	
NA (Optimum Price)	1,721	NA	NA	NA	NA
17.63 (Customary Practice on Market)	1,721	0	3,043,615	599,936,564	0
25 (Actual Offering Price)	1,721	0	3,043,615	599,936,564	0
20.26 (P/E Ratio)	1,721	0	3,043,615	599,936,564	0
33.92 (FCFF)	1,721	0	3,043,615	599,936,564	0
Day 360	I	I	I	I	
13.2 (Optimum Price)	1,721	0.5003	7,855,504	418,719,375	0.5000
17.63 (Customary Practice on Market)	1,721	0	2,696,784	441,185,040	1
25 (Actual Offering Price)	1,721	0	2,696,784	478,463,365	1
20.26 (P/E Ratio)	1,721	0	2,696,784	454,443,967	1
33.92 (FCFF)	1,721	0	2,696,784	523,664,431	1

Table 10Profit Satisfaction  $\beta$ Value of Company C

Selling Pricing Method	Da	ay 60	Day	180	Day	y 360
	Market	Optimum	Market	Optimum	Market	Optimum
Company	Price	Price	Price	Price	Price	Price
Company A	P/E Ratio	Customary Practice on Market	P/E Ratio	FCFF	P/E Ratio	FCFF
Company B	P/E Ratio	Customary Practice on Market	FCFF	P/E Ratio	NA	NA
Company C	Customary Practice on Market	Customary Practice on Market	Customary Practice on Market	NA	Customary Practice on Market	Customary Practice on Market

 Table 11
 Comparison of the Best Valuation Models on Stock Price and Optimum Price

# D. Limitations on a minimum satisfaction

(A) Principle 1: underwriter satisfaction is greater than or equal to the least satisfaction  $\delta$ 

When the issuing company has established standards for least satisfaction; for instance, the satisfaction level of Company A changes from 0.2885 to 0.3887 (interval of 0.378, 0.398), the optimal satisfaction level between the underwriter and the issuing company could not be reached and a difference of  $(\delta = \Delta \mu(z_1)/\mu(z_2))$  occurred. To cope with the requirements for the least satisfaction, the issuing company would require sacrificing the underwriter's profit goal or make use of the fuzzy goals on both sides (Table 12). In the table, the difference in profit satisfaction for both parties changes from  $\Delta = 1$  to  $\Delta = 0.1412$ , which resulted in a reduction in the satisfaction ratio on both sides. Change in profit for both parties  $(-\partial z_2(p,Q_u)/\partial z_1(p,Q_u))$  is 0.0933 with a changed profit membership  $(-\partial \mu_2(z_2(p,Q_u))/\partial \mu_2(z_2(p,Q_u)))$  of 2.3281. This indicates no great impact on the underwriter's profit.

However, the issuing company's profit is affected. For instance, when the underwriter's least satisfaction level is reduced by 0.1, the issuing company's satisfaction level will increase by 0.23. Therefore, both parties may make exchanges in profit satisfaction. For example, when Company A sacrifices \$1 of profit, the underwriter only gets \$0.0933; however, if the issuing company gives away 1% of profit satisfaction, that of the underwriter increases by 2.3281%. Therefore, issuing company would choose profit satisfaction to achieve more direct effect. Take Company A on Day 60 as an example. When both parties play a cooperative game and the underwriter increase profit satisfaction from 0.2887 to 0.3887, the first method will be an adjustment of the offering price by reducing it from \$42.6 to \$16.2 and the second approach is an adjustment of the number of subscription by the underwriter from 1,456 to 1,881, which is also the preferred method. Changes in the Optimum Price and the maximum profit of Company B and C are reported in Tables 13 and 14.

Day 60			
$\beta$ Value of	Change in Profit for	Membership Change	Satisfaction
Underwriter /	Both Parties	for Both Parties	Change Ratio
Issuing Company	$-\partial z_2(p,Q_u)/\partial z_1(p,Q_u)$	$-\partial\mu_2(z_2(p,Q_u))/\partial\mu_2(z_2(p,Q_u))$	$(\delta = \Delta \mu(z_1) / \Delta \mu(z_2))$
0.3887/0.0549	0.0933	2.3281	0.1412
Day 180			
0.6/0.3792	0.0771	1.0301	0.632
Day 360			
0.3961/0.0672	0.0935	2.2952	0.1697

 Table 12 Changes in the Optimum Price and the Maximum Profit of Company A (Principle 1)

 Day 60

 Table 13 Changes in the Optimum Price and the Maximum Profit of Company B (Principle 1)

Day 60			
$\beta$ Value of	Change in Profit for	Membership Change	Satisfaction
Underwriter /	Both Parties	for Both Parties	Change Ratio
Issuing Company	$-\partial z_2(p,Q_u)/\partial z_1(p,Q_u)$	$-\partial\mu_2(z_2(p,Q_u))/\partial\mu_2(z_2(p,Q_u))$	$(\delta = \Delta \mu(z_1) / \Delta \mu(z_2))$
0.5970/0.4034	0.0240	0.9907	0.6757
Day 180			
0.6000/0.4000	0.0241	0.9995	0.6667
Day 360			
NA	NA	NA	NA

Table 14 Changes in the Optimum Price and the	. Maximum Profit of Company C(Principle 1)
Day 60	

$\beta$ Value of Change in Profit for		Satisfaction
Both Parties	for Both Parties	Change Ratio
$-\partial z_2(p,Q_u)/\partial z_1(p,Q_u)$	$-\partial\mu_2(z_2(p,Q_u))/\partial\mu_2(z_2(p,Q_u))$	$(\delta = \Delta \mu(z_1) / \Delta \mu(z_2))$
0.0539	0.9992	0.6667
NA	NA	NA
0.0589	0.9993	0.6414
	Change in Profit for Both Parties $-\partial z_2(p,Q_u)/\partial z_1(p,Q_u)$ 0.0539 NA 0.0589	Change in Profit for Both Parties $-\partial z_2(p,Q_u)/\partial z_1(p,Q_u)$ Membership Change for Both Parties $-\partial \mu_2(z_2(p,Q_u))/\partial \mu_2(z_2(p,Q_u))$ 0.05390.9992NANA0.05890.9993

(B) Principle 2: the satisfaction ratio on both parties  $\delta = \Delta \mu(z_1)/\mu(z_2)$  within the satisfaction interval of the underwriter

When the least satisfaction level of Company A is 0.3887 and the profit satisfaction level is limited to 0.379, 0.399, the satisfaction ratio on both parties is  $\Delta$ =0.1412. As this is inconsistent with the satisfaction interval of the underwriter, the underwriter may accept reduced satisfaction to seek a more suitable profit satisfaction level for both parties (see Table 15). When Company A changes the least satisfaction from 0.3887 to 0.3508 and both parties exchange profit or profit membership, the final profit satisfaction ratio will be 0.3911, which is consistent with the subjective range for the least satisfaction for the underwriter. Changes in the optimum Price and the maximum profit of Company B and C are listed in Tables 16 and 17.

Table 15 Changes in the Optimum Price & the. Maximum Profit of Company A (Principle 2) Day 60

Day 00					
$\beta$ Value of Change in Profit		Membership Change	Satisfaction Change		
Underwriter / Both Parties		for Both Parties	Ratio		
Issuing Company	$-\partial z_2(p,Q_u)/\partial z_1(p,Q_u)$	$-\partial\mu_2(z_2(p,Q_u))/\partial\mu_2(z_2(p,Q_u))$	$\delta = \Delta \mu(z_1) / \Delta \mu(z_2)$		
0.3508/0.1372	0.0972	2.4243	0.3911		
Day 180					
0.6/0.3792	0.0771	1.0301	0.632		
Day 360					
0.3664/0.1310	0.0960	2.3566	0.3573		

Table 16 Changes in the Optimum Price and the Maximum Profit of Company B (Principle 2)

Day 00					
$\beta$ Value of Change in Profit for M		Membership Change	Satisfaction Change		
Underwriter /	Both Parties	for Both Parties	Ratio		
Issuing Company	$-\partial z_2(p,Q_u)/\partial z_1(p,Q_u)$	$-\partial\mu_2(z_2(p,Q_u))/\partial\mu_2(z_2(p,Q_u))$	$\left(\delta = \Delta \mu(z_1) / \Delta \mu(z_2)\right)$		
0.6157/0.3843	0.0240	0.9914	0.6242		
Day 180					
0.6159/0.3841	0.0241	0.9995	0.6236		
Day 360					
NA	NA	NA	NA		

Table 17 Changes in the Optimum Price and the. Maximum Profit of Company C(Principle 2) Day 60

$\beta$ Value of Change in Profit for		Satisfaction Change			
Both Parties	for Both Parties	Ratio			
$-\partial z_2(p,Q_u)/\partial z_1(p,Q_u)$	$-\partial\mu_2(z_2(p,Q_u))/\partial\mu_2(z_2(p,Q_u))$	$(\delta = \Delta \mu(z_1) / \Delta \mu(z_2))$			
0.0539	0.9992	0.6106			
Day 180					
NA	NA	NA			
Day 360					
0.0590	0.9993	0.6206			
	Change in Profit for Both Parties $-\partial z_2(p,Q_u)/\partial z_1(p,Q_u)$ 0.0539 NA 0.0590	Change in Profit for Both Parties $-\partial z_2(p,Q_u)/\partial z_1(p,Q_u)$ Membership Change for Both Parties $-\partial \mu_2(z_2(p,Q_u))/\partial \mu_2(z_2(p,Q_u))$ 0.05390.9992NANA0.05900.9993			

# 5. Conclusions

The purpose of this study is to explore how the profit maximizing, optimum offering price and profit maximizing goals are computed for both parties using pricing models when the underwriter and the issuing company maintain a cooperative relationship. The underwriter and issuing company profit functions are linearly programmed with the fuzzy variables rationalized through an interactive fuzzy game programming method for solutions to the maximum profit and optimum offering price on both parties. Both parties may exchange profit to promote satisfaction when the underwriter sets up the least satisfaction level. Six conclusions are reached in this study.

First, a fuzzy regression is designed by inputting specific values and output fuzzy numbers to be more consistent with the uncertain relationship of independent and dependent human interventions in reality. As the profit function is a probability function, underwriters may exercise judgment in compliance with their experience.

Second, for estimation of the lottery odds and handling fees in a bear market, the average lottery odds is 22.86%, which is much higher than the mean of 8.45%. This shows that the economic situation impact is extreme. The regression results are more consistent with the bear market. The average handling fee is 1.78%, which is far below the minimum processing fee specified by the SFC and indicates that

underwriters take a conservative attitude on handling fees.

Third, the optimum prices calculated at different offering periods (Day 60, Day 180 and Day 360) for Company A are \$42.6, \$45.8 and \$45.3, respectively. All prices are below the actual offering price of \$60. The offering price of \$63.07 is based on the P/E ratio approach. If Company A maintains the offering price of \$60, the underwriter may increase the number of subscription shares up to 3256, which results in a profit satisfaction level  $\beta$  value of 0.5 for both parties. The optimum prices calculated at different offering periods (Day 60 and Day 180) for Company B are \$141.4 and \$355, respectively. The actual offering price of \$278 range between them. As the actual offering price of Company B is based on the P/E ratio approach with a 10% off of \$364.3 as the reference price and Company B subscribe to 100% of its' shares, the offering price may be adjusted to the optimal price of \$355 to reduce the difference in profit satisfaction levels on both parties with the  $\beta$  value of 5. The optimum prices calculated at different offering periods (Day 60, Day 180 and Day 360) for Company C are \$14.75, and \$13.2, respectively. This is below the actual offering price of \$25. However, the actual offering price is above the prices calculated using three valuation models. This indicates that the underwriter overestimates the offering price. The solution for Company C is to take hedge/risk The common satisfaction levels for the three companies on aversion measures. 60-Day are 0.2887, 0.4981 and 0.5000 respectively. Of the prices from three pricing models, all are below the maximum satisfaction except the customary market practice on 60-Day and 360-Day for Company A and 180-Day for Company B. Differences in satisfaction levels exist between the underwriters and the issuing companies. From the perspective of the actual offering prices, the satisfaction levels for the three sample companies are overestimated, while those for the underwriters are insufficient. Profit or the level of satisfaction can be exchanged during the offering price negotiation process.

Fourth, the optimum price must be lower than the market value. Prices based on customary market practices are significantly more underestimated than those calculated using other pricing models. Thus, prices based on the customary market practices are closer to the optimum prices. When the market value is below the offering price and underwriters expect a bear market, prices based on customary market practices might be adopted to increase profit or reduce loss. The P/E ratio approach is used to price Companies A, B and C. The underwriters suffered capital losses from the stocks they subscribed to, which is inconsistent with profit maximizing pricing. Although the underwriters possess the most professional judgment and information on the optimal prices; they may not obtain what they desire because they do not have bargaining power comparable to that of the issuing companies. If the P/E ratio of a similar industry is overestimated, prices based on the P/E ratio approach had the highest possibility for overestimation in the three valuation models. This might cause the issuing companies to increase the offering price in pursuit of their own interest.

Fifth, for Companies B and C, the maximal satisfaction level for the optimum price is close to 0.5, which indicated that both the underwriters and public companies considered the profit satisfaction of the other while negotiating offering prices. However, the prices from the three pricing models and actual offering prices are different from the optimum prices. This reveals asymmetrical information or legal limitations on the number of underwriting or subscription shares by the underwriters. Thus, the maximal satisfaction on both parties could not be achieved.

Finally, a comparison of changes in profit and sensitivity to satisfaction between the underwriters and issuing companies can serve as a reference for exchange to promote the level of satisfaction for both underwriters and issuing companies.

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